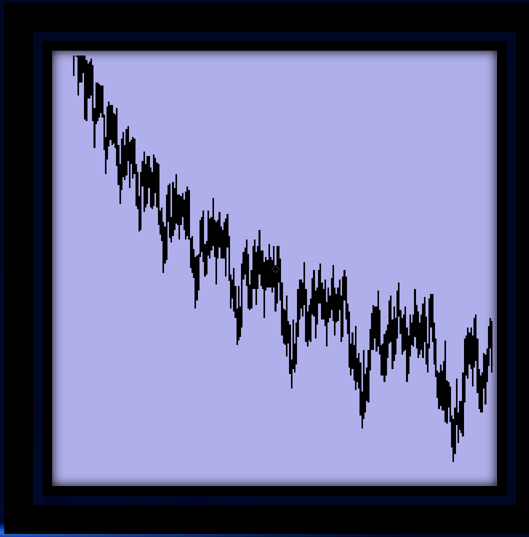
NADIA SYEDA

Senior from University At Buffalo (Pursuing B.A. degree in Mathematics and Economics)

Collaborated with Dr. Chris Yuen, University At Buffalo

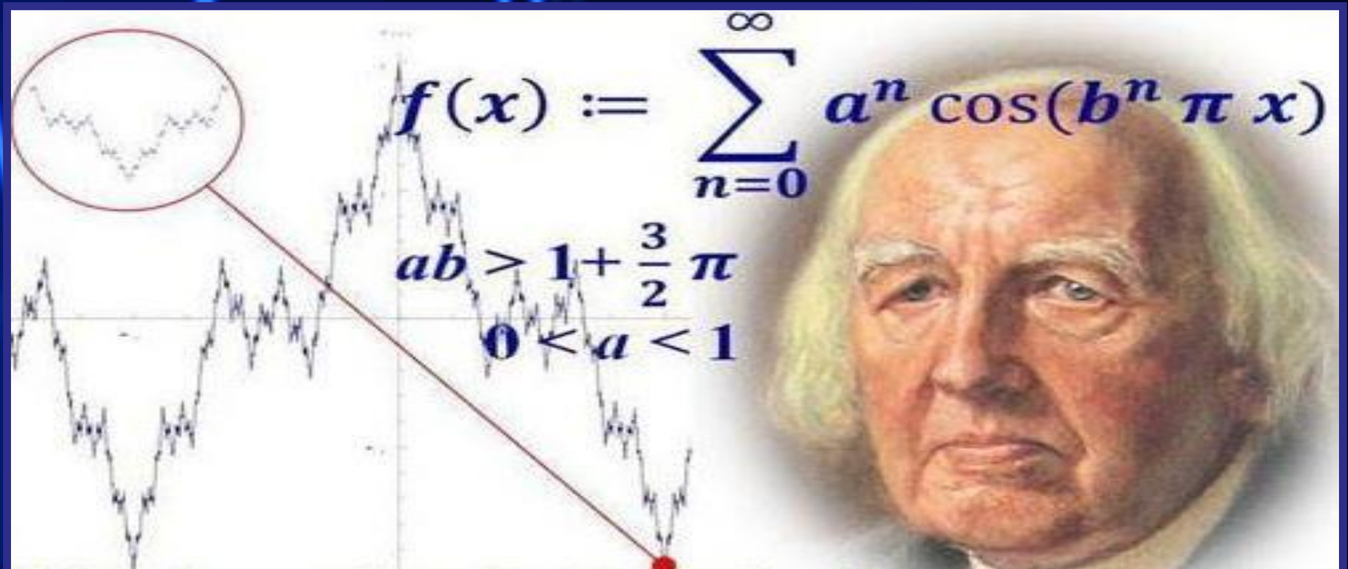
Wonderworks Behind The Weierstrass Function: It's Applications And Impacts To The Instruction In Calculus



The Man Behind The Weierstrass Function

Karl Theodor Wilhelm Weierstrass

The Father Of
Modern Analysis



The Man Behind The Weierstrass Function

born

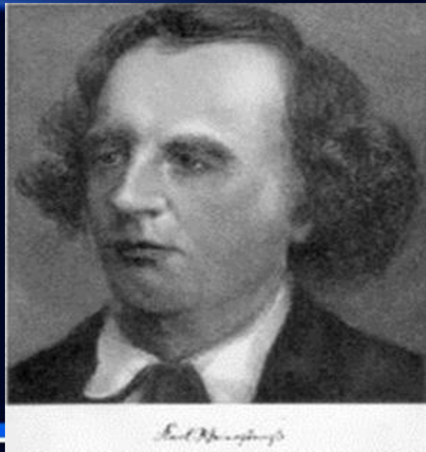
- October 31, 1815

Enrolled in Law school at the University of Bonn, Germany

- 1834

Published first paper on “The Theory of Abelian Functions”

- 1854



The Man Behind The Weierstrass Function

Published second paper on “Abelian Functions” and became full time professor at the University of Berlin

- 1856

Presented the Weierstrass Function to the Königliche Akademie der Wissenchaften

- 1872

The Man Behind The Weierstrass Function

Became the Knight Of The Order, “Pour le Mérite” Category of Arts and Science, highest honor given in Germany

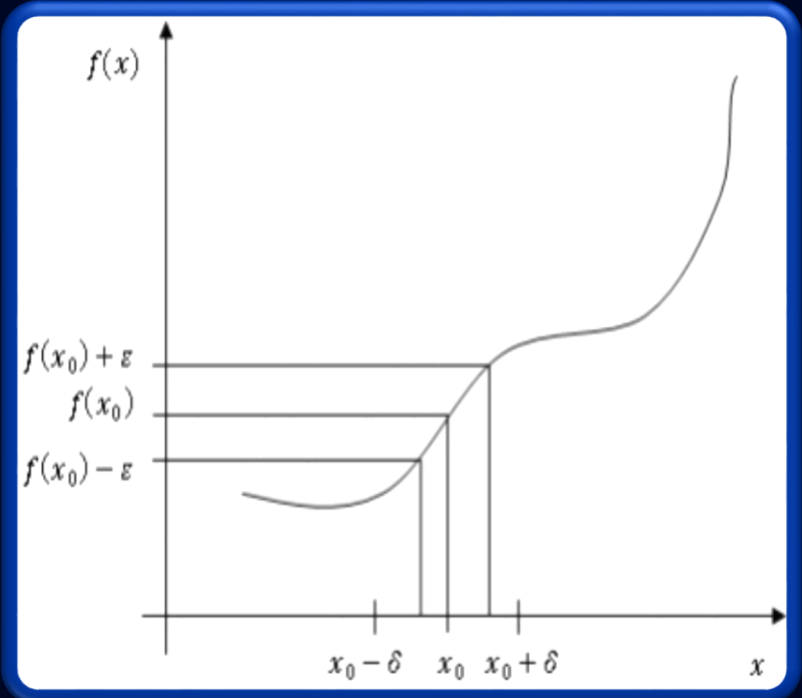
- 1875

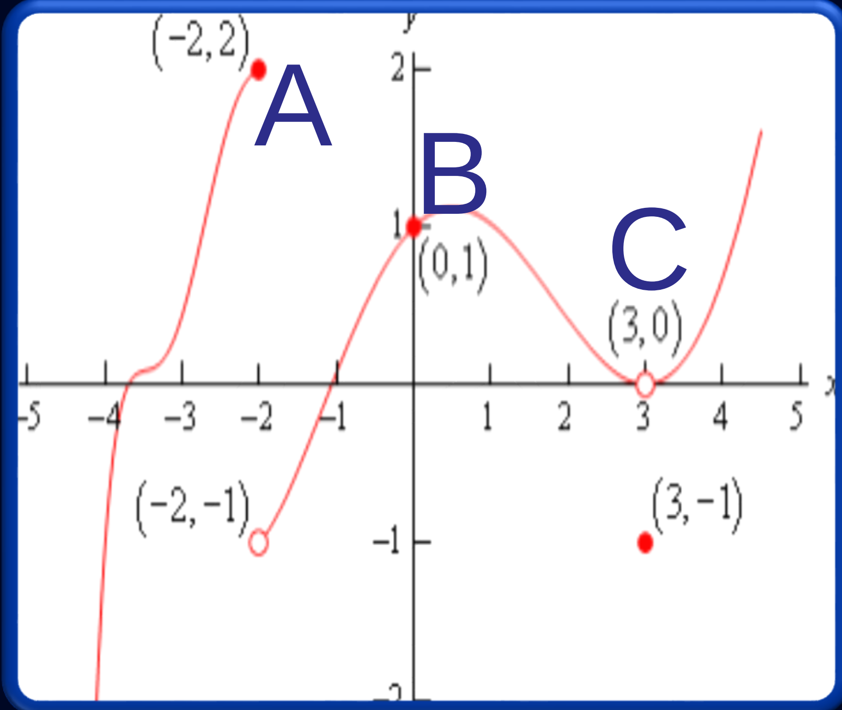
Weirstrass passed away leaving his notes and ideas to his devoted students

- 1897

Definition of Continuity

The function f is said to be continuous on A $\leftrightarrow \forall\, x_0 \in A, \forall\, \varepsilon > 0\ \exists\ \delta > 0, \forall x \in A$ $|x - x_0| < \delta \rightarrow |f(x) - f(x_0)| < \varepsilon$ (where $A \subseteq \mathbb{R}$ and $f: A \rightarrow \mathbb{R}$ will be a real valued function on A.)

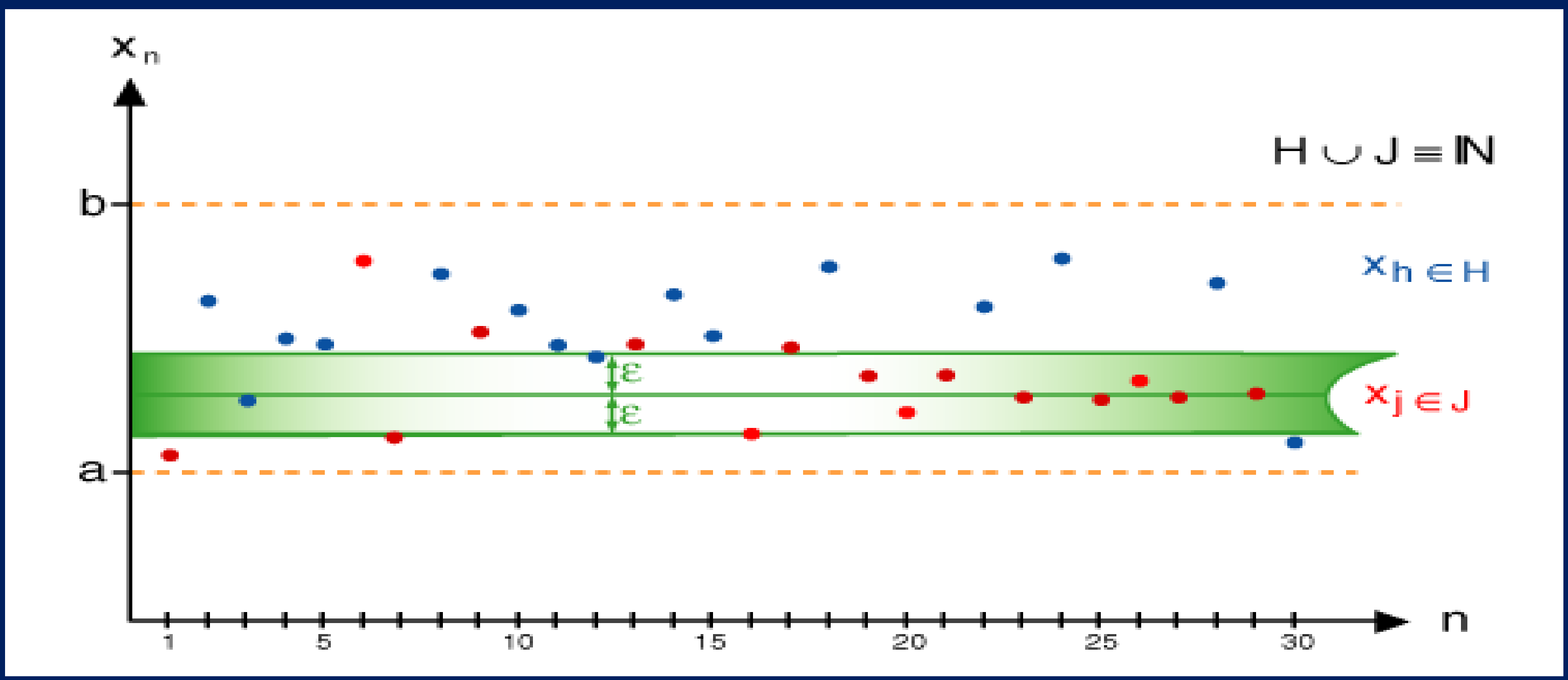




Great Achievements: The Bolzano-Weierstrass Theorem

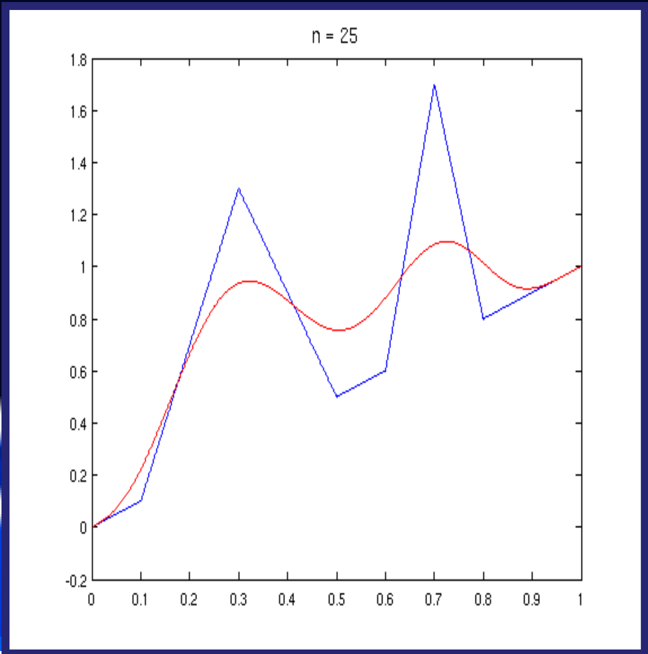
Every bounded sequence (x_n) has a convergent subsequence (x_{n_k})

where $n, k \geq 1$

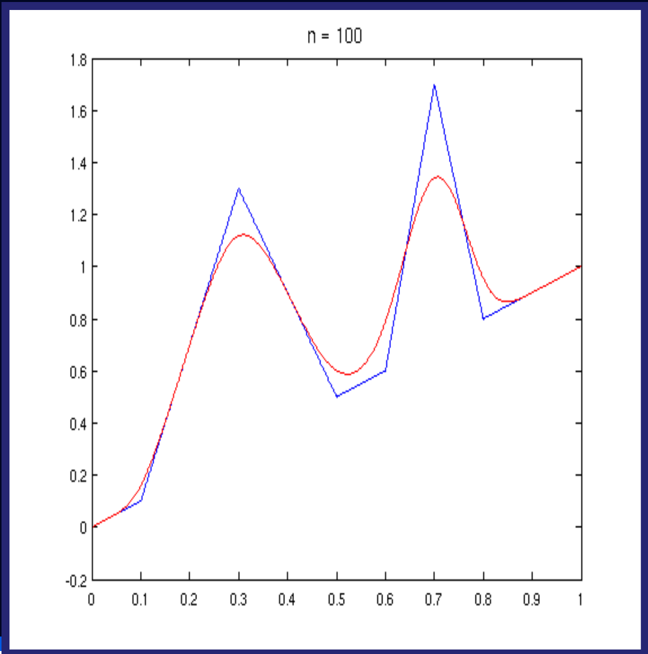


Great Achievements: Weierstrass Approximation Theorem

- If f is continuous at real valued functions on $[a, b]$ and if any $\varepsilon > 0$ is given $\exists$ a polynomial p on $[a, b]$ such that $|f(x) -$



n = 25

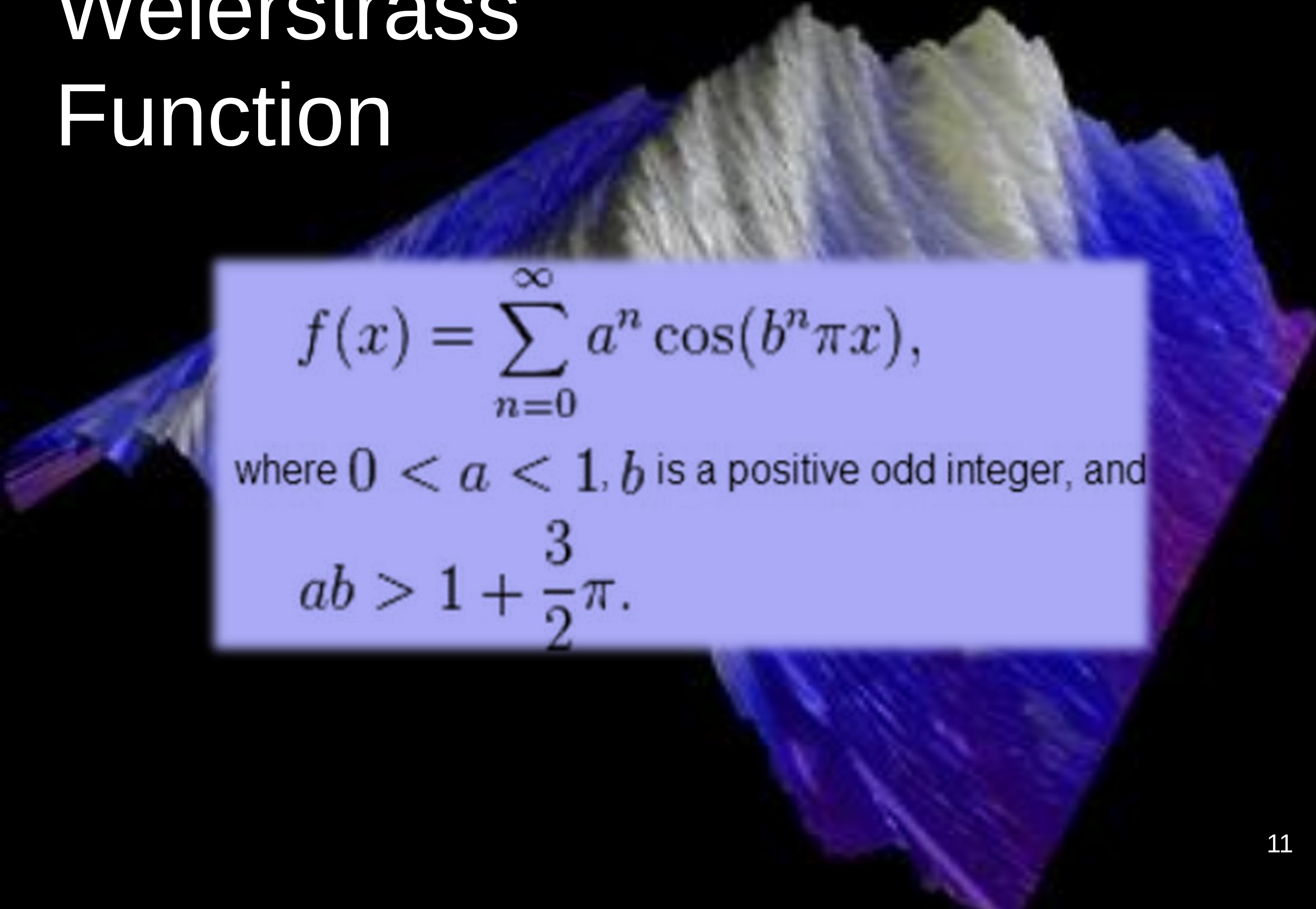


n= 100

Great Achievements: Weierstrass M-Test

- Let (f_n) be a sequence of functions defined on A {interval of some set}
- Suppose (M_n) is a sequence of positive numbers such that $|f_n(x)| < M_n$ for each n on A where $\sum_{n=0}^{\infty} M_n < \infty$
- Suppose $\sum_{n=0}^{\infty} M_n$ converges then $\sum_{n=0}^{\infty} f_n(x)$ converges uniformly on A to the function as well

Weierstrass Function

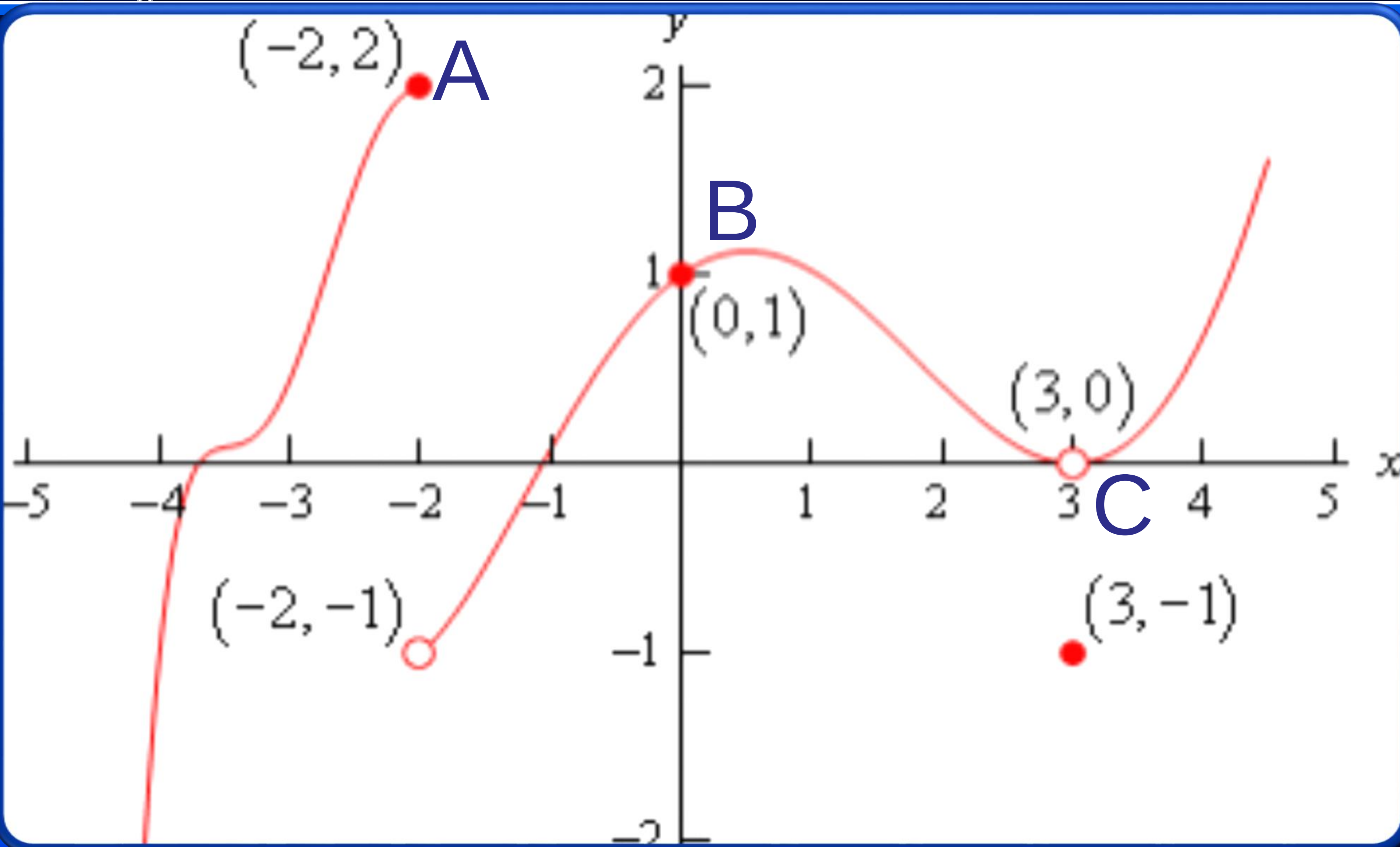


$$f(x) = \sum_{n=0}^{\infty} a^n \cos(b^n \pi x),$$

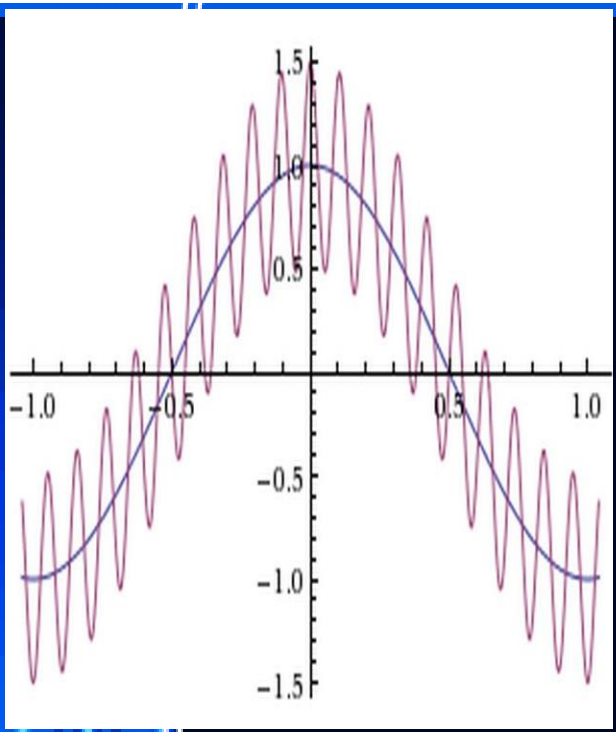
where $0 < a < 1$, b is a positive odd integer, and

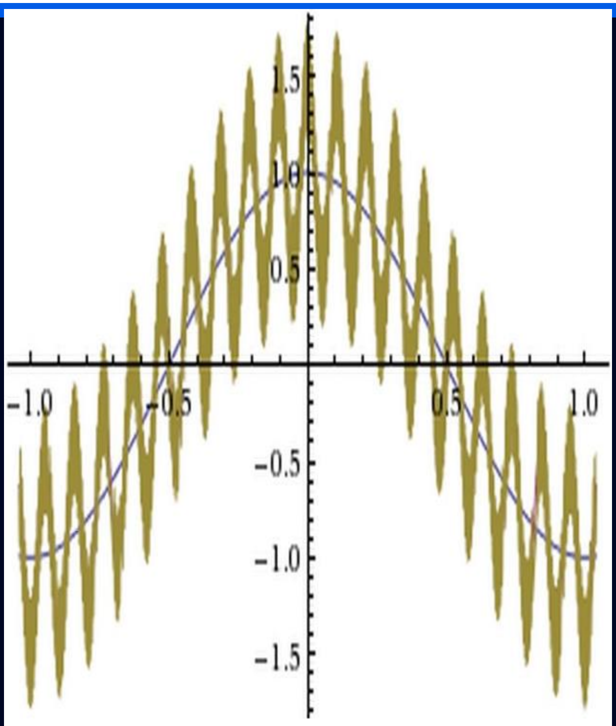
$$ab > 1 + \frac{3}{2}\pi.$$

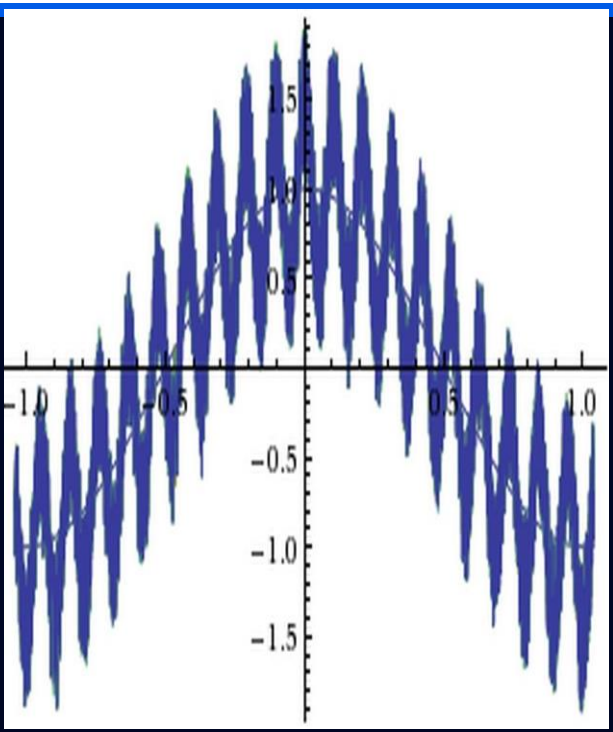
Differentiable or Non-Differentiable



Properties Of The Weierstrass Function







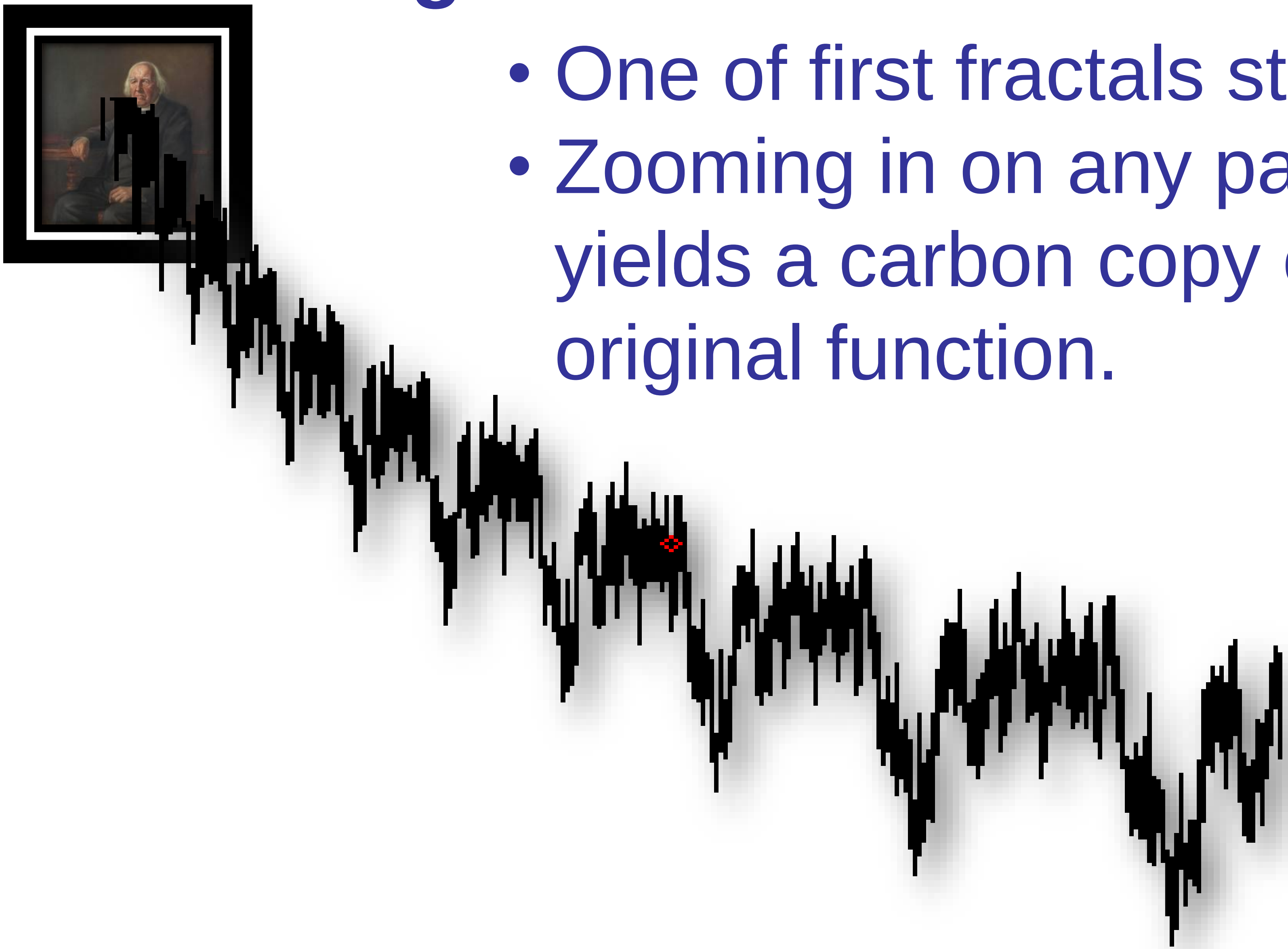
$$\sum_{n=0}^{1} a^n \cos(b^n \pi x)$$

$$\sum_{n=0}^{2} a^n \cos(b^n \pi x)$$

$$\sum_{n=0}^{10} a^n \cos(b^n \pi x)$$

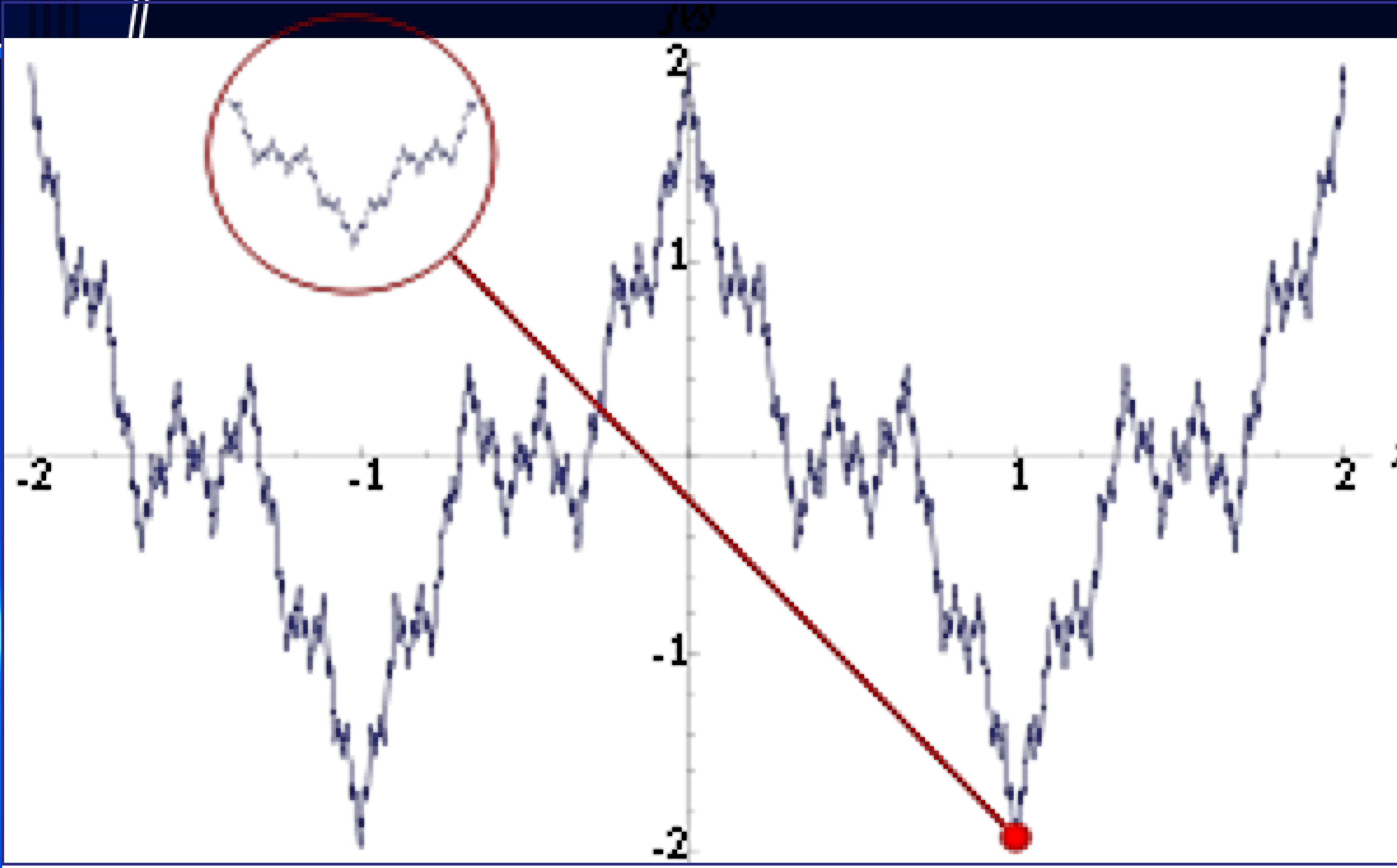
$\forall\ n, x \in A$ continuous everywhere differentiable nowhere.

Weierstrass Function as a Rising Fractal

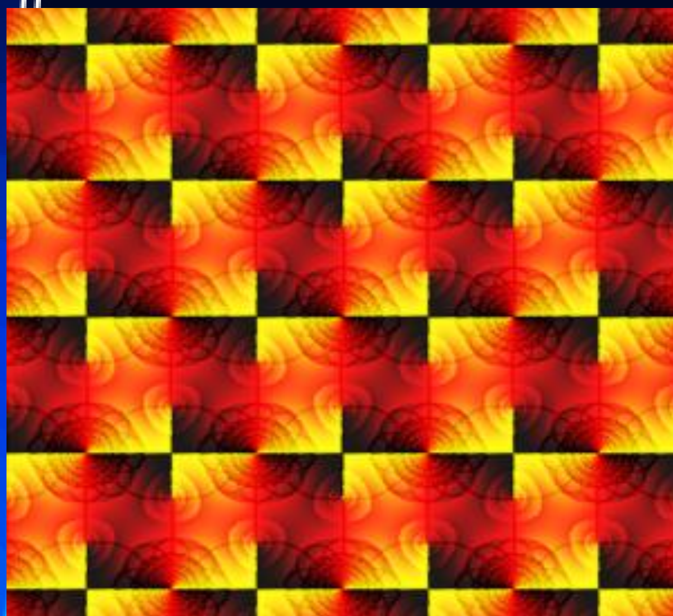


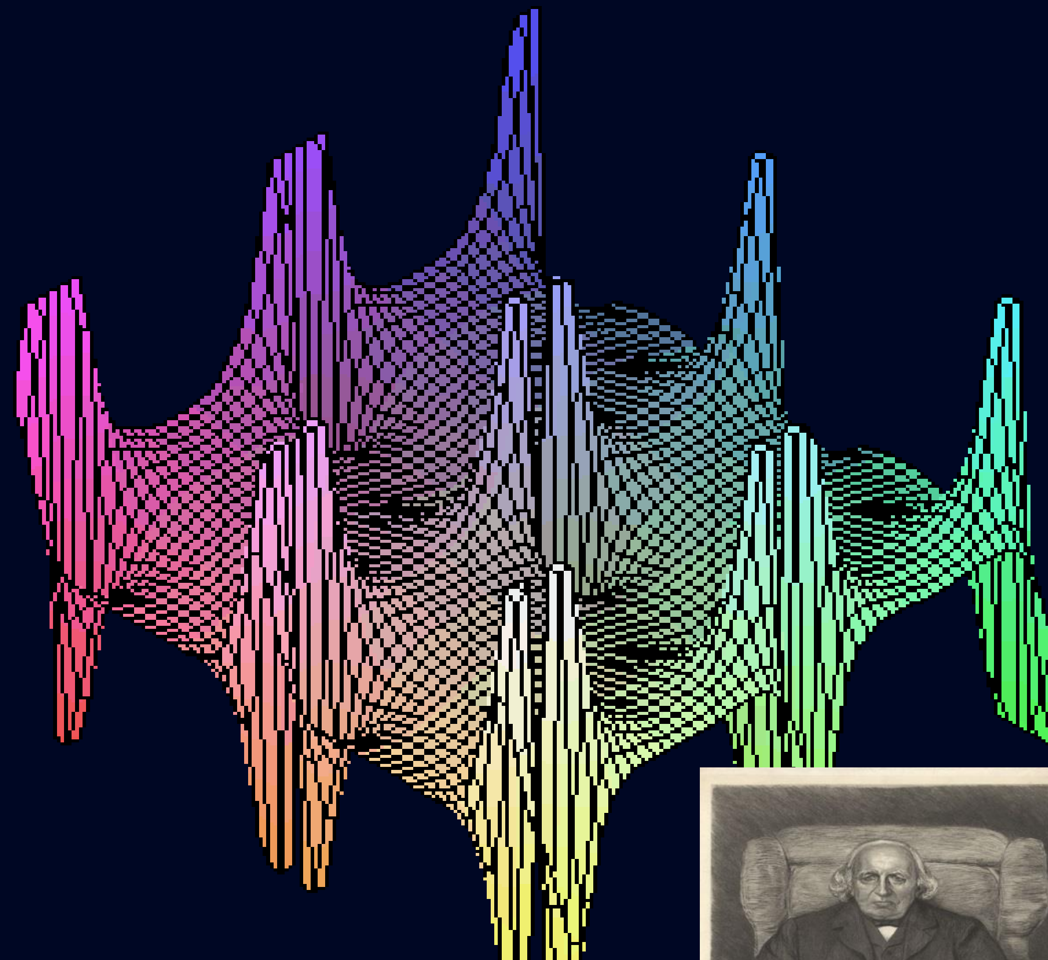
- One of first fractals studied.
- Zooming in on any part yields a carbon copy of the original function.

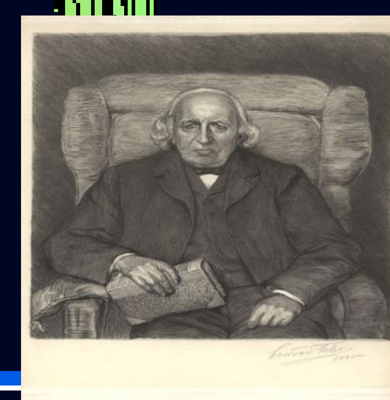
Zoomed in on $x = 1$



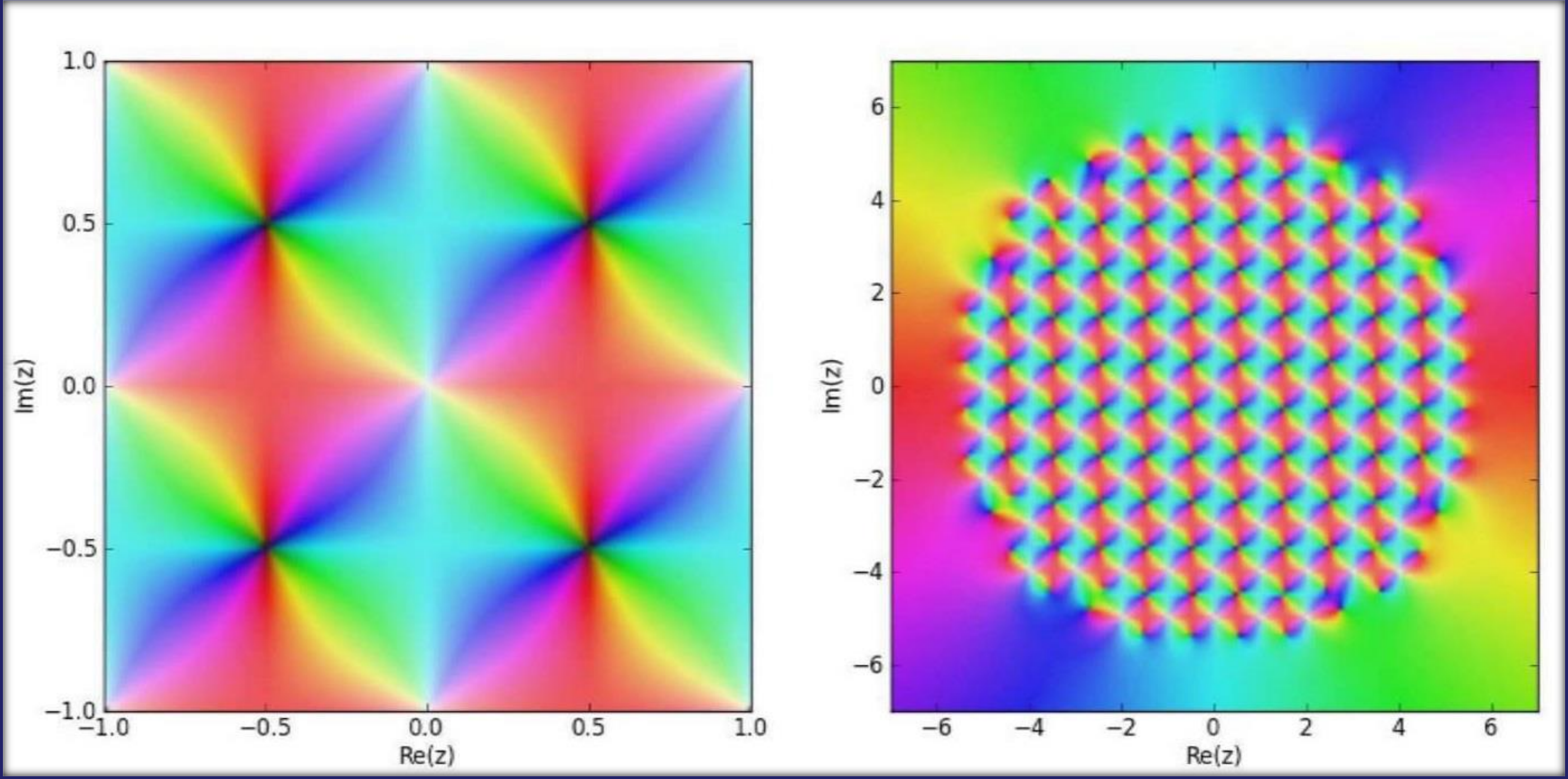
Weierstrass Elliptic Function



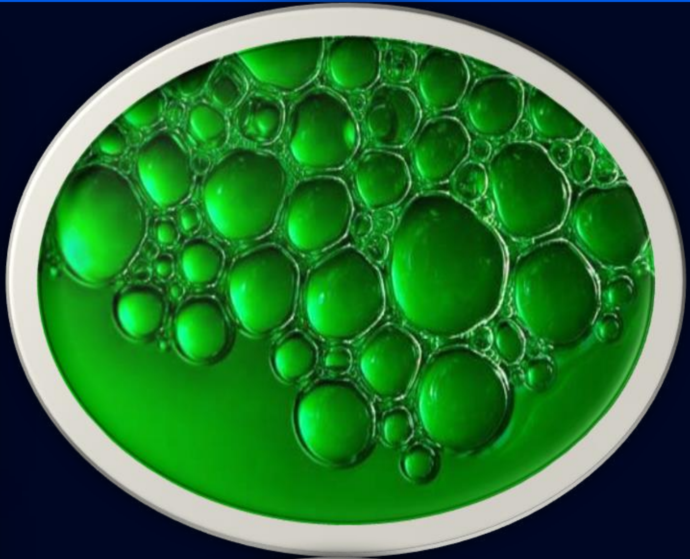




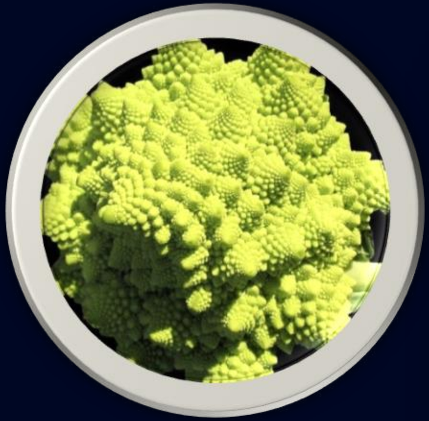
Weierstrass Elliptic Function



Natural Fractals

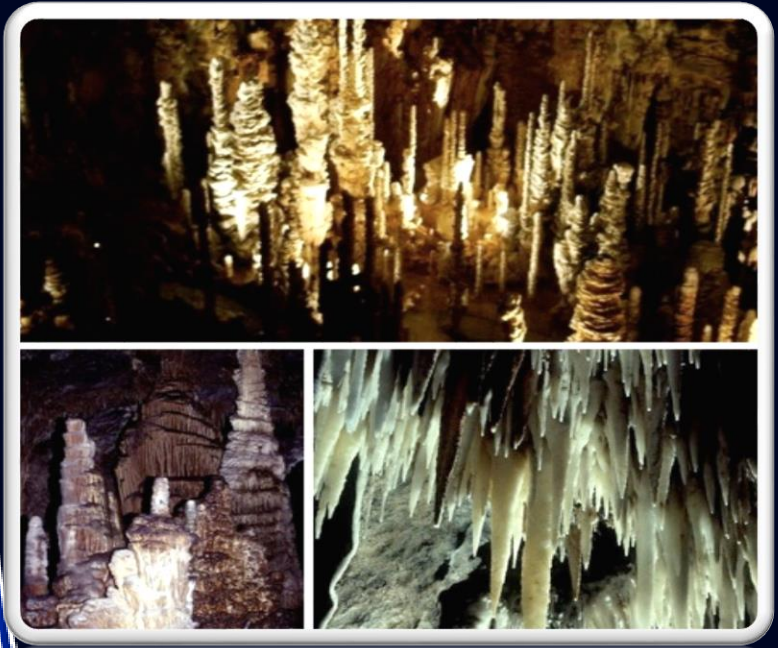
Floral Iterative Blooms

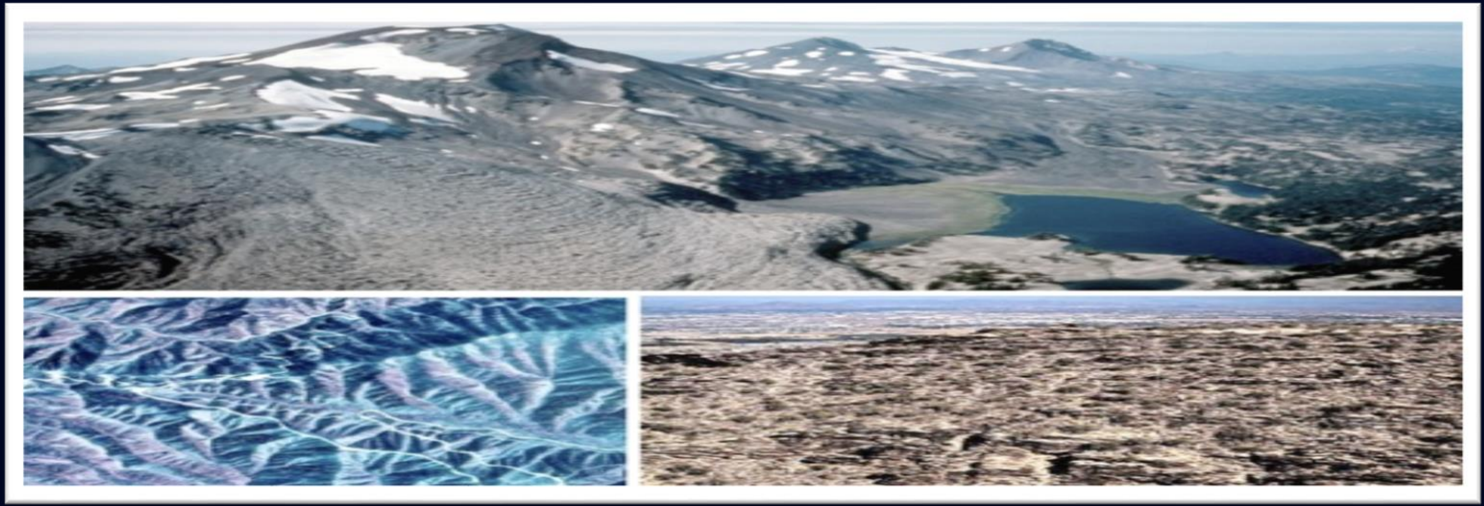


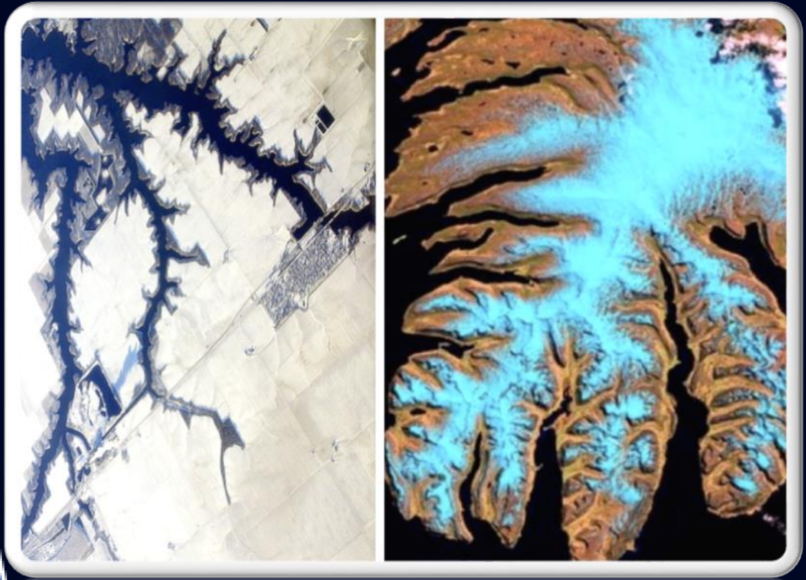
Fractals in Light of Physics

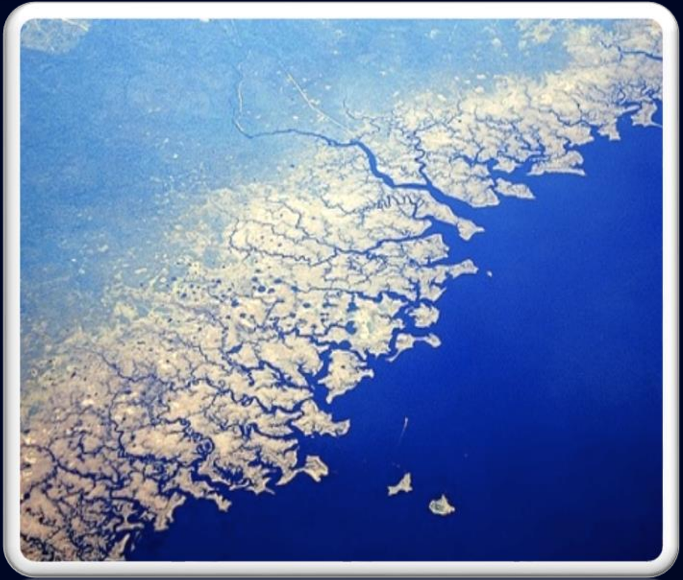
Natural Fractals



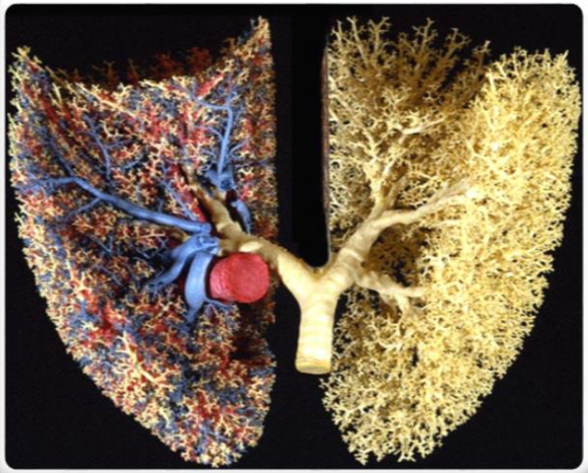
Fractal Architecture

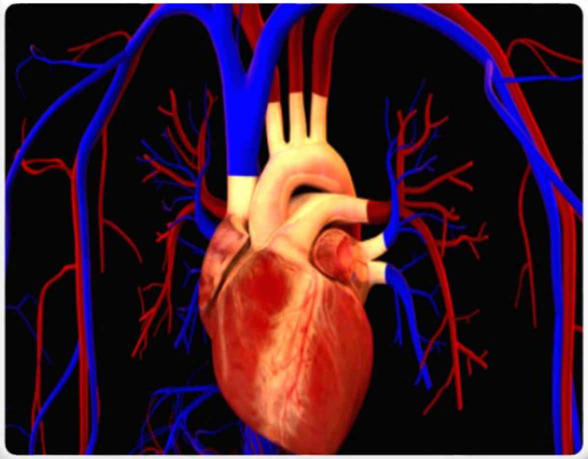






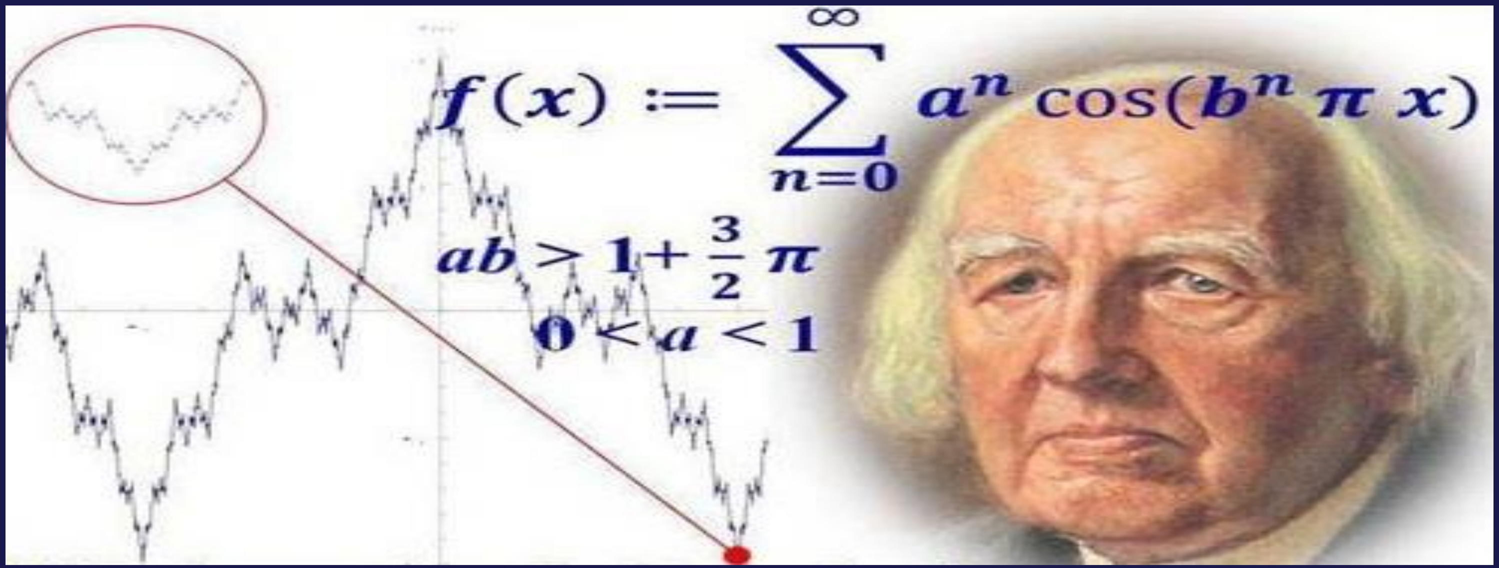
Fractal Forms in Physiology





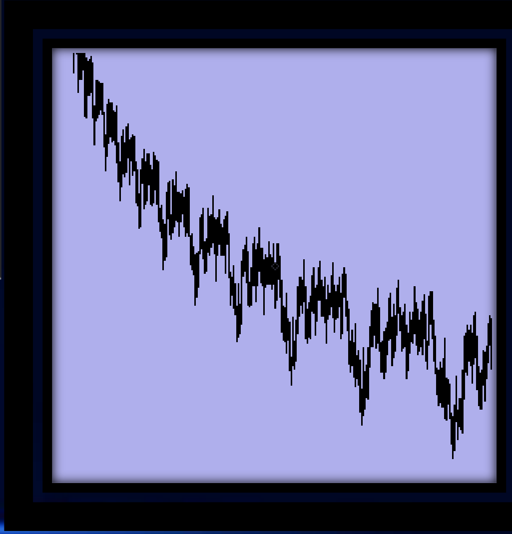
In The End……..

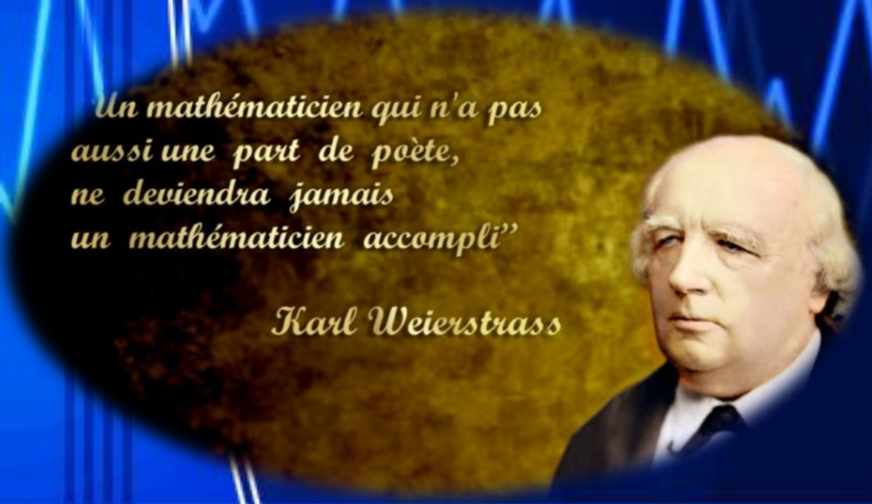
- Kept unfinished papers in a large white wooden box which was lost while on a trip in 1880
- Age 75 he retired; spending life confined to a wheelchair



"It is true that a mathematician who is not also something of a poet will never be a perfect mathematician.”

~ *Karl Theodor Wilhelm Weierstrass*





QUESTIONS

COMMENTS

CONCERNS

The End

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