Single Inflation Model with Radiated Corrections

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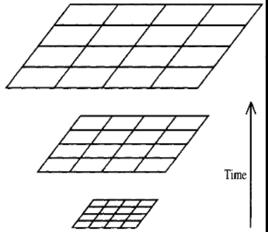
Introduction

When the Universe expands, every point in space moves away from each other much like dilating a coordinate basis. The interval between spaces is known as the scale factor, a(t), and is used to describe the behavior of the expansion.

The purpose of this research is to test a particular model of single inflation using a numerical analysis. This model includes corrections that should be accounted for given by physical phenomenon that are suspected to be present at the time of inflation.



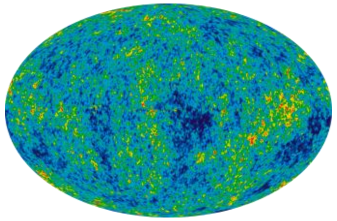
Equations

The Friedmann Equation describes the time evolution of the expansion of the Universe via the scale factor a(t).
(left)

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3} - \frac{k}{a^3}$$

On the other hand, the continuity equation describes the time mass/energy density of the Universe.

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + P)$$



3 Big Problems

Horizon Problem
- The Cosmic Microwave Background Radiation (left) can be thought of as a relic of old light with temperature at 2.725K. This contradicts the idea of disjoint patches of the Universe where there is no energy transfer; furthermore, the universe appears to be the same throughout (Homogeneity), and the same everywhere we look (Isotropic).

Flatness Problem
- On a grand scale, the Universe appears to be flat. At the very early Universe, conditions had to be so precise that the contribution of curvature would need to be less than 16 orders of magnitude smaller than the density of radiation, making this a "fine-tuning" problem.

Magnetic-Monopole Problem
- At the very early universe, some Grand Unified theories predict that there were many hot and dense magnetic monopoles; however, only magnetic dipoles have been found in nature.

Cosmological Constant

The Inflationary model suggests that if a reservoir source of energy called the Cosmological Constant (Λ), are plausible solutions to the previous problems. According to theory, space was once a near point, but as time progressed, the high dominance of the cosmological constant caused rapid expansion in fractions of a second. If every localized horizon originated from a mutually smaller space, it can be expected that the temperature and curvature between two close patches are near equal. Inflation also accounts for the Flatness Problem, by considering that the Universe experienced different phases of cosmological constant, radiation, and matter dominance. Since the Universe, was at such hotter and denser temperatures, the existence of magnetic monopoles could arise from phenomena that occur outside the energy levels in the Standard Model.

Single Inflation and Slow Roll

The Inflationary period lasted from about 10^{-36} s to 10^{-32} seconds after the Big Bang and is a period where the scale factor is accelerating. Namely, $\ddot{a}(t) > 0$, which implies that expansion is accelerating. As the universe began to expand, the overall potential decreases. In this particular model, Inflation is driven by a "single" scalar potential, V(φ), that is approximated as a classical scalar field and depends on inflaton, φ.

$$V(\varphi) = \frac{1}{2}\kappa^2\varphi^2 - \lambda\varphi^4 ln\varphi$$

In the Slow Roll approximation, the quantity $\ddot{a}(t)$, is approximated to be very small and, thus, negligible. An analogy can be made to a ball slowly rolling down a hill of potential. Equivalently, there are slow roll parameters ε, and η, which depend on the first and second derivatives of the scalar potential. ε can be thought of as energy density, but both parameters are effectively dimensionless. Also, r is the tensor-to-scalar ration and n_s is the scalar spectral index and are related to the Slow Roll Parameters. The final parameter, Δ_R^2, is the curvature perturbations. Ultimately, the Slow Roll approximation is a computational choice that allows us to approach the differential equations pertaining to Inflation.

Parameters

- $\varepsilon = \frac{1}{2}\left(\frac{\frac{dV}{d\varphi}}{V}\right)^2$
- $\eta = \frac{\frac{d^2V}{d\varphi^2}}{V}$
- $r = 16\varepsilon$
- $n_s = 1 - 6\varepsilon + 2\eta$
- $\Delta_R^2 = \frac{1}{12\pi^2}\frac{V^3}{\left(\frac{dV}{d\varphi}\right)^2}$

Method

The numerical analysis was focused on solving two partial differential equations with two unknowns. The two unknowns in question are the scalar spectral index and the curvature perturbations. In order to conduct this, an open source computer algebra system known as Maxima was used. The partial differential equations were a result of applying Multivariable Newton's method on two linearly independent equations. We approached the problem by setting both equations into a linear system and simultaneously solving both using matrix algebra.

Computational Analysis

We apply Taylor's theorem to two functions of two variables, $f(x,y)$ and $g(x,y)$. Setting $f(x,y) = 0 = g(x,y)$ gives the following equations:

$$f(x,y) = 0 = f(a,b) + \frac{\partial f(a,b)}{\partial x} + \frac{\partial f(a,b)}{\partial y}$$

$$g(x,y) = 0 = g(a,b) + \frac{\partial g(a,b)}{\partial x} + \frac{\partial g(a,b)}{\partial y}$$

In matrix form they are represented as,

$$-\begin{pmatrix} f(a,b) \\ g(a,b) \end{pmatrix} = \begin{pmatrix} \frac{\partial f(a,b)}{\partial x} & \frac{\partial f(a,b)}{\partial y} \\ \frac{\partial g(a,b)}{\partial x} & \frac{\partial g(a,b)}{\partial y} \end{pmatrix} \begin{pmatrix} x-a \\ y-a \end{pmatrix}$$

If we set matrix A to

$$A = \begin{pmatrix} \frac{\partial f(a,b)}{\partial x} & \frac{\partial f(a,b)}{\partial y} \\ \frac{\partial g(a,b)}{\partial x} & \frac{\partial g(a,b)}{\partial y} \end{pmatrix}$$

Then our solution becomes

$$\begin{pmatrix} x-a \\ y-a \end{pmatrix} = -A^{-1}\begin{pmatrix} f(a,b) \\ g(a,b) \end{pmatrix}$$

The following code includes the iterative do loop used to compute the system of differential equations. This was provided by our advisor, Professor Matthew Civiletti.

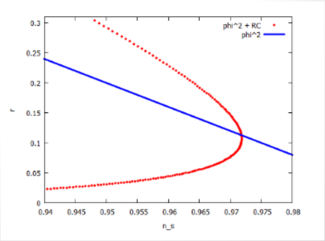
```
F:matrix([-DeltaRSq(phi)],[-rSolve(phi)])$
D:matrix([diff(DeltaRSq(phi),phi),diff(DeltaRSq(phi),lambda)],
[diff(rSolve(phi),phi),diff(rSolve(phi),lambda)])$
Dinv:invert(D)$
deltax:Dinv.F$

for n: 1 thru Nmax do

(g1del:float(ev(deltax[1][1],phi:g1,lambda:g2)),
g2del:float(ev(deltax[2][1],phi:g1,lambda:g2)),

g1:g1+g1del,
g2:g2+g2del,
```

The following plot, also provided, predicts the value for n_s to be approximately 0.92 which is close to theory.



References

Liddle, A. *An Introduction to Cosmology*. Chichester: Wiley, 2004. Print.

"Tests of Big Bang: The CMB." *WMAP Big Bang CMB Test*. N.p., n.d. Web. 06 Jan. 2016