RECTANGULAR FRACTION MODELS: AN EXPLORATION OF THE IMPACT OF CONCRETE MATHEMATICS MANIPULATIVE USE IN THE FIFTH GRADE

by

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A Master's Thesis/Project Capstone
Submitted in Partial Fulfillment
of the Requirements for the Degree of
Master of Science in Education
Curriculum and Instruction in Inclusive Education
Department of Curriculum and Instruction
State University of New York at Fredonia
Fredonia, New York

May 2015

State University of New York at Fredonia
Department of Curriculum and Instruction

CERTIFICATION OF THESIS/PROJECT CAPSTONE WORK

We the undersigned, certify that this project entitled RECTANGULAR FRACTION MODELS: AN EXPLORATION OF THE IMPACT OF CONCRETE MATHEMATICS MANIPULATIVE USE IN THE FIFTH GRADE by RACHAEL H KIBLER, Candidate for the Degree of Master of Science in Education, Curriculum and Instruction in Inclusive Education, is acceptable in form and content and demonstrates satisfactory knowledge of the field covered by this project.

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ABSTRACT

The purpose of this mixed methods study was to explore how implementation of a new, researcher-developed *simple improvised manipulative* (SIM) impacted 5th graders in an urban, Common Core-aligned classroom. The *Rectangular Fraction Model*, a SIM created with two overlapping pieces of transparent plastic, was tested through performance of this experiment. This research sought to answer the following two central questions: How does implementation of a SIM, the Rectangular Fraction Model, impact 5th grade students' math achievement in a mathematics class at an urban Chautauqua County elementary school in Western New York? How does use of a concrete representation affect students' conceptual understanding of abstract material as taught through the Common-Core aligned EngageNY curriculum? The researcher was interested in two areas of possible impact on student learning; student achievement measured by a formal assessment and student understanding of abstract materials evaluated through use of an interview and questionnaire. Twelve students participated in the study; they were placed in heterogeneous control and experimental groups. The results indicate that although students in the experimental group scored better on the posttest and appeared to have a better understanding of the concept taught, the difference between the control and the experimental group was not statistically significant. Thus, the use of SIM is not more effective than the traditional teaching approach. However, student responses indicate an interest in using this type of intervention material, and further research should be conducted on the impact of SIMs in the mathematics classroom.

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Introduction

Exploring ways of helping students to learn has been a heavily researched topic. In the area of mathematics education, many students struggle with understanding and applying abstract concepts. Recent data suggests that the United States has a faltering achievement rate in mathematics (Education Next, 2010; National Center for Education Statistics, 2011). The Program for International Student Assessment (PISA) is an international survey designed to collect achievement data on the competencies of 15-year olds internationally. According to their 2012 data on mathematical achievement, the United States ranked 31^{st} of 65 participating countries, trailing far behind leading nations (PISA Report, 2012). With technological advancements and the increase in STEM-related careers, there is a push to better student mathematics achievement in promotion of college and career readiness.

Traditionally, teachers have sought to improve student achievement on assessments and promote conceptual understanding of abstract topics through use of manipulatives. Many teachers and researchers have observed the impact of using such interventions, and many studies have included the design of unique interventions, ranging from paper handcrafted geometry tools to fraction pieces (Aburime, 2007; Andrews, 2004; Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Mistrella, 2000). Overall, the literature has shown that students tend to benefit greatly from the use of concrete learning tools, or manipulatives, when learning mathematics (Puchner, Taylor, O'Donnell, & Fick, 2008, Sherman & Bisanz, 2009).

The importance of studying the impact of manipulatives on student achievement and understanding of mathematical concepts was the research problem. For the purpose of this study, the term *manipulative* is defined as any supplemental learning tool designed for implementation within a lesson to further student conceptual understanding of abstract concepts. *Concrete manipulatives* are defined as physical objects which can be moved or arranged to demonstrate a mathematical idea. A *SIM*, or *simple improvised manipulative*, is any handcrafted, teacher devised concrete manipulative which may be constructed from a variety of accessible materials (i.e., paper, cardboard, wood).

SIMs are frequently created by classroom teachers using readily available materials, and thus can be convenient and cost effective tools for learning (Andrews, 2004). The purpose of this study was to explore the impact, if any, teacher-created concrete manipulative use has on student achievement in mathematics. Specifically, this study explored the use of a teacher-created SIM, Rectangular Fraction Models (a physical enrichment of an area model using pieces of clear plastic transparency sheets imprinted with rectangle models), in a fifth grade setting as an alternative means of teaching an EngageNY lesson. Furthermore, the researcher studied the effect of this particular SIM item on student's mathematics achievement, when using it to teach a lesson that is aligned to the Common Core Learning Standards.

Implementation of the Common Core Learning standards across America has changed the way teachers prepare curriculum and have impacted teaching methods. In New York State, EngageNY has prepared a series of curriculum modules for implementation in the K-12 general-education classroom. Many of the lessons designed by the State Education Department incorporate learning activities and opportunities for exploratory learning and a more concrete learning approach to abstract concepts. For instance, Module 4 of the fifth grade EngageNY math curriculum introduces students to manipulating fractions through use of pictorial diagrams.

Although some research has been conducted on the effectiveness of the new learning standards, there appears to be a deficiency in the literature exploring the implementation of manipulatives in coordination with EngageNY lesson plan use. The use of manipulatives to teach an EngageNY lesson to fifth graders was studied, and the effect of use of the Rectangular Fraction Model was explored for possible correlation with student achievement. The classroom teacher does not currently implement a concrete Rectangular Fraction Model in her teaching of Module 4, and was intrigued to see how it might affect student learning.

As both an educator and new teacher, the researcher is very interested in adding to her teaching tools any items proved beneficial in helping students achieve. This research was significant in that it sampled a new population of participants and tends to provide this population with a physical means of understanding abstract concepts through implementation of easy to create, accessible and inexpensive learning tools. The researcher aims to share the findings of the effectiveness of the implemented SIM with fellow new teachers.

In summary, this study sought to answer the following questions:

- How does implementation of a SIM, the Rectangular Fraction Model, impact 5th grade students' math achievement in a mathematics class at an urban Chautauqua County elementary school in Western New York?
- How does use of a concrete representation affect students' conceptual understanding of abstract material as taught through the Common-Core aligned EngageNY curriculum?

The hypothesis was that student achievement will be higher with manipulative use as an intervention, and that conceptual understanding would be heightened through the use of the concrete learning tools.

This study was significant in that it contributed to the literature on manipulative use through exploration of the impact of a researcher-created SIM on 5^{th} grade student achievement. Specifically, this research incorporated the EngageNY modules-a deficit area of research with this topic. Through this mixed methods study, the deficiency in the literature was addressed through collection and analysis of data which ultimately proved the research hypothesis correct. Through comparing results from Common-Core Engage NY lesson-aligned pretest and posttest data, the researcher presents to future readers data-supported evidence of the impact of manipulative use. This mixed methods study gives pre-service teachers and colleagues insight into the impact of incorporating manipulatives into EngageNY lessons.

Literature Review

Introduction

For decades, American students have struggled with mathematical achievement (National Center for Education Statistics, 2011). Since the year 2000, the International Organization for Economic Co-operation and Development (OECD) has operated the Program for International Student Assessment (PISA). Since its start, PISA has documented achievement rates in mathematics through collection and statistical analysis of a sample of students that represents the full population of 15-year-old students in 65 participating countries and educational systems. Data is collected once every three years, and achievement is documented though assigning students to a proficiency level. To reach a particular proficiency level, a student must correctly answer a majority of items at that level (PISA Report, 2012). Students were classified into mathematics literacy levels according to their scores, which range from 0-1000.

As reported in the 2012 Program for International Student Assessment report, student achievement in the United States as measured by performance tests is low; 52.2% of 15 year-old American students performed at a Level 2 or lower (PISA, 2012). This is a high percentage when compared to other OECD countries; in 2012, only 22.5% of 15 year-olds throughout OECD countries performed at a Level 2 or lower. Students performing at a Level 2 or below are performing below average and thus are considered as low achievers. For clarification, a Level 2 is defined as a score greater than 420.07 and less than or equal to 482.38; hence students performing at a Level 2 are scoring at approximately a 42-48% on assessments (out of 100% achievement). These results are publically published by the United States Department of Education to inform readers.

The use of mathematics manipulatives, defined as learning tools to facilitate math instruction and conceptual understanding of abstract concepts, has been heavily researched in the field. Although some research has suggested that manipulatives result in varied achievement outcomes (Carbonneau, Marley, & Selig, 2013), others have supported that implementation of mathematics manipulatives in the classroom resulted in a positive trend in promoting mathematical understanding (Sherman & Bisanz, 2009). Thus educators have tended to promote the student benefits of using evidence-based learning tools by sufficiently well-trained teachers (Puchner et al., 2008).

This review explores the mathematics achievement rate in the United States and addresses how teachers use both purchased and self-created (SIM) manipulatives to enrich their instruction, exploring the effects of using concrete learning tools to teach abstract concepts. Few studies have been conducted to provide insight regarding the effectiveness of manipulative use in the common core curriculum. The purpose of this quantitative study is to address this deficit in the literature through exploring the potential impact of manipulative use in teaching the Common Core in the classroom.

For this literature review, the databases accessible for use by Fredonia students were utilized. The researcher relied on the Academic OneFile, Academic Search Complete, ERIC Database and EBSCOhost library databases to locate scholarly, peer-reviewed journal articles. Google Scholar was also used, but to a lesser extent. Key search terms for locating relevant articles were: *math, mathematics, education, childhood, elementary, primary, secondary, children, adolescents, manipulatives, concrete, abstract, learning, tools,* and *realia.* Articles relevant to the current study were cited in APA format and included in the reference list for this study.

Mathematics Achievement in the United States

National and international assessments have documented the difficulty of mathematics for many students (PISA Report, 2012). The 2011 National Assessment of Educational Progress results indicated that 60% of fourth-grade and 57% of eighth graders struggled in math, performing at or below proficiency levels on standardized assessments (National Center for Education Statistics, 2011). This achievement gap has carried into the upper grades as well, as detailed by a report published by Harvard's Education Next program. In 2005, only 6% of eighth grade students achieved at an 'advanced' level in mathematics; in a study of 2009 graduates, only 8% of students scored at an advanced level (Education Next, 2010). The achievement gap differed from state to state; regardless, students nationwide have shown a tendency to struggle with mathematical concepts.

As a nation the United States has fallen behind globally in mathematics achievement rates, as indicated by Center Research (Carbonneau et al., 2013). Traditionally, when compared to peers internationally, American students regularly have underperformed their global peers on mathematics assessments (Education Next, 2010). In recent years American students appear to be becoming stronger math students, as reported by the Trends in International Math and Science Study (TIMSS), which has collected and analyzed assessment data on 4^{th} and 8^{th} grade math achievement since 1997. From 2011 data, TIMSS found United States 4^{th} graders to be achieving in mathematics at a level among the top 15 education systems in mathematics (TIMSS Report, 2011). TIMSS shows that the average mathematics score of U.S. 4th-graders (541) was higher than the international TIMSS scale average (500).

Although these recent figures suggest a trending positive increase in American students' mathematical aptitudes, data shows many of our nation's children struggle with math (PISA

Report, 2012). Ultimately, poor nationwide mathematics achievement has created difficulty for students who are considering STEM (science, technology, engineering and mathematical science) careers (National Center for Education Statistics, 2008). It has been suggested that through better achievement rates and conceptual understanding of mathematical ideas, students can be better prepared for STEM-field careers (Education Next, 2010).

Teacher's Role in the Use of Mathematics Manipulatives to Facilitate Learning

The use of concrete learning objects, or math manipulatives, is being studied for its effects on student achievement. The interest in the possibly positive effects of manipulatives on mathematics achievement is rooted in long-held beliefs by popular psychology that a child's brain develops the most through play with concrete objects (Burns & Hamm, 2011). It is a teacher's level of comfort in using manipulatives that affects classroom benefits (Joyner, 1990). Often, teachers must be taught the benefits of using manipulatives to further learning (Uribe-Flórez, Lida, & Wilkins, 2010). In the middle grade levels, a teacher may fall into the trap of using manipulatives as classroom supports or diversions from abstract lessons where teachers lack content knowledge (Moyer, 2001).

Other teachers report having flourished through use of manipulatives to enrich the teaching of mathematics (Uribe-Flórez & Wilkins, 2011). The effectiveness of the manipulative is contingent on the teacher's level of familiarity with the manipulative and the context in which it is used (Belenky & Nokes, 2009). For instance, learning fractions is facilitated through the use of manipulatives, be it fraction tiles or pie pieces. Brown and McNeil (2009) found that teachers often used manipulatives ineffectively or incorrectly due to poor training, and suggested teachers use their own judgment when implementing a manipulative. Belenky and Nokes (2009) found that elementary students tended to learn quicker when the manipulative was a realistic, accurate representation of the concept applied, hence when learning fractions most student preferred learning with the fraction pieces as they better conveyed the fractions as part of the whole.

The implementation of manipulatives can be beneficial across the grade levels, provided teachers are readily prepared to use them. One thought stressed throughout the literature is that a teacher should not implement interventions and manipulatives if uncomfortable with their use (Bouck & Flanagan, 2010; Brown, McNeil & Glenberg, 2009; Moyer, 2001). Researchers have highlighted professional development in using math manipulatives as key in a successful implementation (Puchner et al., 2008). Teachers must devote time to profession development and communicate with other teachers their satisfaction or discomfort with a given manipulative; earnestly exploring manipulatives is a task worth the time and effort in seeking classroom results (McNeil & Jarvin, 2007).

Implementation of Manipulatives

Educators have used manipulatives as a form of intervention in the classroom, with use ranging from computer programs to concrete realia to graphing calculator applications (Ganesh & Middleton, 2006; Widmer & Sheffield, 1994). Virtual computerized learning aids have shown increasingly popular in school settings (Sarama & Clements, 2009). Software programs which digitize concrete objects are helpful in having students expand their thinking in number sense. According to Sarama and Clements (2009), students who worked through computer software programs to improve math skills experienced greater mathematical accuracy and precision in calculations. Furthermore, students became more analytical and were able to think more symbolically (Sarama & Clements, 2009). Ngan Hoe and Ferrucci (2012) explored the effectiveness of fraction software used with primary students. They found that the software greatly increased achievement scores while developing a greater understanding of fractional parts (Ngan & Ferrucci, 2012).

In further investigations of the use of virtual manipulatives in the classroom, Lee and Chen (2010) found students felt greater confidence in their mathematical abilities when using virtual tools. Other studies have also looked at the use of Interactive Whiteboard technologies and their effectiveness in the mathematics classroom (Wu-Yuin, Jia-Han,Yueh-Min, &Jian-Jie, 2009). Mildenhall and Swan (2008) recommended that - although there were benefits of interactive whiteboards, teachers must be willing to use new methods, including that requiring technology integration.

Manipulatives can also take the form of concrete, physical objects in the classroom learning environment. Number sense can be built using counters, buttons, or unifix cubes (Carbonneau et al., 2013). Arithmetic can be improved through the use of manipulatives focused on presenting addition, subtraction, multiplication and division in concrete terms (Sherman & Bisanz, 2009). The Rekenrek, an abacus-type tool, has proven to help students with special needs visualize addition and subtraction (Tournaki, Young, & Kerekes, 2008). Fractions have successfully been explored through fraction strips and fraction pieces (Butler et al., 2003). Burns and Hamm (2011) concluded that students who worked with manipulatives understood symmetry better than peers who did not. Geometry can be explored through use of geoboards, geometric shapes, geo-mirrors, graphing mats and shapes blocks (Salend & Hofstetter, 1996). Manipulatives can even take the form of folded paper (Aburime, 2007) and other handcrafted creations (Mistrella, 2000). Manipulatives can be store-bought or inexpensive do-it-yourself projects, as in the case of a Kindergarten teacher who made a math center game from an old cookie sheet (Andrews, 2004). Manipulatives can be created from pre-existing classroom materials. SIMs, or simple improvised manipulatives, are an affordable means of differentiated instruction in the classroom environment (Aburime, 2007).

In the secondary setting, students work with graphic organizers, charts and diagrams to develop a better conceptual understanding of mathematics (Allsopp, 1999). According to Allsopp, there should be a focus on the use of "evidence-based practices" which boost mathematical comprehension through peer feedback, think-pair-share activities and strong modeling. "Representational Instruction" was the term used by Allsopp to define teaching with use of manipulatives to further promote conceptual understanding (Allsopp, 1999). One such topic which has shown potential for strong representational instruction is in the area of statistics and averages (Baker & Beisel, 2001). According to Baker and Beisel's work (2001), when given the opportunity to further explore the concept of average through computer programs and concrete manipulatives with 22 fourth through sixth grade students, the students excelled in both achievement and understanding. Concrete manipulatives may also help students understand rounding and estimation (Bohan & Shawaker, 1994).

A rewarding aspect of appropriate manipulative use has been the potential to elicit student self-discovery of abstract concepts (Lapp, 1999). Eighth-grade students show higher motivation when given a learning tool, as Allsopp (1999) argued the case when mnemonics and think-aloud were utilized in instruction. Through the simple activity of "drawing" one's solution to a problem, a student develops abstract thinking abilities (Allsopp). A research study conducted in the Southwestern United States showed great achievement rates among middle school students who used fraction manipulatives to better conceptually understand improper fractions (Butler et al., 2003). The study showed that students benefited from concrete representations of abstractions. This particular study was conducted in an urban district, but positive reports of manipulative use have been shown in various school settings and with varying sample populations. A 1996 practitioner article discusses the how students learn to reason, problem solve and communicate mathematically through use of geomirrors and Cuisenaire rods in the inclusive setting (Salend & Hofstetter, 1996).

Classroom Outcomes Related to Mathematics Manipulative Use

Although some studies on manipulatives have shown inconclusive evidence that manipulatives are beneficial (Carbonneau et al., 2013), many studies in recent years have supported a growth in student achievement from pre-test to post-test for students using manipulatives (Baker & Beisel, 2001). Students who use manipulatives have become stronger problem solvers (Belenky & Nokes, 2009). Students who were asked to think in non-symbolic terms and use mathematical manipulatives displayed greater mathematical aptitude and accuracy, compared to peers who were asked to think in symbolic terms (Sherman & Bisanz, 2009). Students were shown to better analyze numbers in abstract ways through use of concrete learning tools; these tools have helped solidify comprehension of topics (Burns & Hamm, 2011).

Many studies have looked at computer-based software programs and their effectiveness in supplementing traditional textbook learning; however, as stated in the research of Burns and Hamm, there may be little to no difference between using a virtual manipulative or concrete manipulative; both prove mutually effective (Burns & Hamm, 2011).

Mathematics Manipulatives and the Common Core

The New York State Common Core modules (marketed as EngageNY) have supplemented lessons with the use of concrete learning materials. Manipulatives are both provided by teachers and embedded within individual lessons through the use of supplemental handout sheets, which direct students to create a relevant learning tool. For instance, the third grade EngageNY Lesson 12 of Module 3 material directs students to create a set of fraction strips using a supplemental handout (attached to the teacher-accessible document).

Teachers have explored the use of manipulatives to teach to the Common Core curriculum, largely through use of technology. Educreations and Evernote are two of many virtual whiteboard manipulatives that students can use to explore different strategies when solving complex math problems (Hillman, 2014). Other downloadable applications for iPads can help students practice Common Core aligned material; Hillman suggests Numbers League, Scootpad and Skoolbo Core Skills are applications worth downloading to help students practice their math skills. DreamBox Learning has expanded online resources to include digital curriculums complete with interactive virtual manipulatives that are Common Core Aligned (Learning and Leading with Technology, 2012).

As the Common Core is a recent curriculum guideline which many schools are still adopting, teachers are exploring ways to implement manipulatives. Although literature exists on the use of virtual manipulatives with the Common Core, there is less literature on the use of concrete manipulatives to teach lessons aligned with the Common Core. This study seeks to address the deficiency in the literature by exploring the potential effects of using Common Core created concrete learning materials to teach mathematics in the elementary setting.

Methodology

Overview of Research Method

This study used a mixed methods design; it studied the impact of a particular intervention on a target population through research and investigation. The researcher created heterogeneous groupings (an even number of low, average, and high level achievers in each group) for a control and experimental group. Formal assessment data was collected through administration of a pre and posttest to both the control and experimental group. These data were statistically analyzed using Microsoft Excel and MiniTab software. A questionnaire was given and a focus interview conducted to gather qualitative data. These data were coded and analyzed.

Participants and Setting

The site of this study was an elementary school in an urban area of Chautauqua County in Western New York. The specific school district was chosen for its diverse student population and favorable accessibility. In 2012, the elementary school had a total enrollment of 185 students (110 male, 85 female), and there were 20 students in the fifth grade (New York State Report Card, 2013). This school had a high poverty rate in 2012, with 81% or 150 of the 185 elementary students economically disadvantaged. Free lunch was found to be prevalent (75% of the elementary students received free lunch, and 6% qualified for a reduced price lunch).

Concerning ethnicities, 55% of students identified as Latino/Hispanic, 32% identified as white, and 12% of the student population was comprised of other ethnicities. Many of the students were multilingual, with Spanish being the most common language spoken by students. In 2012, 19% of the elementary population had limited English proficiency and received supports. Students with documented disabilities comprised 4% of the school population.

Moreover, as a school that frequently works with the College and is heavily supported by pre-service teacher endeavors, it seems the school fit the experimental design of the study well.

As this study explored the use of manipulatives in the mathematics setting with fifth graders, it was relevant to look at assessment data for this age group. All 20 fifth grade students were given the 2013 New York State mathematics assessment, aligned to Common Core State Standards. No students with disabilities took the exam. Ten of the students were Hispanic/Latino, eight were White, and two were Black. No Hispanic students received scores indicating proficiency in mathematics (as denoted by a score of Level 3 or 4). The majority (90%) of Hispanic students scored a 1, and 10% scored a 2. Six students were documented as having limited English Language proficiency. Of all fifth grade students tested, 65% scored the lowest score possible on the exam, Level 1. Thirty percent of this population performed at a Level 2. It is important to note that class demographics vary from year to year. Although these figures do not represent the achievement levels of the participants of this study, they point out a general characteristic of students in this school-many struggle with mathematics.

For the purpose of this study, 24 fifth grade participants (age 10-11) were selected; this sampling was a convenience sampling. Eight were female and 16 were male. Four of these students, two male and two female, received AIS mathematics supports in both a push-in and pull-out setting. Fourteen were of Hispanic ethnicity, and the remaining ten were of White or mixed ethnicity. The students were divided into two heterogeneous groups, based on present achievement levels. There were two above grade level students, three at grade level students, and seven below grade level students in each group. One group (Group C) was assigned as the control group, and the second group was the experimental group (Group E).

The population and setting for this study were chosen with intent; these factors represent a convenience sampling due to researcher's preference to perform the study in this specific school and classroom setting with the participating cooperating teacher and chosen students. The researcher has worked with the selected participants in both the general education and AIS setting as a substitute teacher, and felt her comfort with the classroom and knowledge of student needs aided the administration of the experiment. This classroom participated in center activities during the daily 30 minute Math Extension, and the researcher saw this as an opportune period in which to conduct her research.

Study Design

This study aimed to explore the effects of implementation of a SIM manipulative, Rectangular Fraction Models, in a diverse classroom setting. This study looked at the impact of the Rectangular Fraction Model on student achievement. Through conduction of an experiment, the researcher explored how concrete manipulative use impacted fifth grade student achievement when learning about multiplying fractions as supported by the EngageNY Module 4 lessons.

Due to the nature of this study and its design, this study was best described as a mixed methods design. In this study, participants were not chosen at random. The assignment of students to their groups (C or E-control or experimental) was done through consultation with the classroom teacher and through careful consideration of student achievement levels to make the groups balanced. The researcher matched student present achievement levels to create heterogeneous groupings; there was an even number of low, average, and high level achievers in each group.

SIMs, or simple improvised manipulatives, utilize the forethought of the proactive teacher in developing learning tools to enrich student learning. In teaching the multiplication of basic unit fractions, EngageNY suggests students draw an area model; students are to show through drawing a gridded rectangle the product of two unit fractions. This concrete example of an abstract concept aims to help students understand what it means to multiply two fractions. A concern with any hand drawn scrap work, particularly with younger students and those with special needs, is in the accuracy of the depiction of abstract concepts. For instance, although a fraction is defined as an equal part of a whole, students often fail to demonstrate this definition through drawings of fraction models due to a) developing fine motor skills and b) a lack of true conceptual understanding.

In this study, the researcher introduced the self-created Rectangular Fraction Model SIM, a set of two overlapping pieces of transparent plastic with congruent, equally partitioned rectangles printed on the clear transparencies. These sheets were dry-erase, which allowed students to shade in the fractional factors. Students shaded and labeled each factor, and then placed the sheets together to see what portion of the whole (the rectangle) is shaded. This was intended to help students find the product of two fractions. The use of the SIM focused on the same concrete application of an abstract concept as used in the problem set, but extended its use to a hands-on, interactive learning tool that promoted a clear, uncluttered pictorial representation.

The design of this study was action research learning with a pretest-posttest design. The dependent variable was student achievement in the area of multiplying fractions measured by pre- and post test. The independent variable of the study was the SIM researcher-created math manipulative, Rectangular Fraction Models. The study was conducted over the course of two week, with six days spent in the field collecting data.

On day one of data collection, students were given a pretest comprised of ten example questions on multiplying fractions. Questions were presented as both equations and word problems. For three days, students explored multiplying fractions in their assigned group during center time, a 30 minute mathematics instruction extension period. All student work was completed independently.

On day one of instruction, students in both control and experimental groups worked through the same lesson using identical Lesson 13 of Module 4 Problem Sets. Students in Control Group C stayed in the classroom with the teacher, who guided students through the problem set using only the worksheet and no manipulatives. Students calculated answers through completing pictorial representations of the area model. In experimental Group E, students worked with the researcher in the cafeteria to complete the problem set; however, the Rectangular Fraction Models was introduced as a tool for students to use. Students were encouraged to explore the multiplication problems through the use of the SIM. Although the SIM was provided to offer enrichment of the material, students were still expected to show their work on the problem set page through drawing a rectangular fraction area model.

For the next three days (days two through four of the study) students explored multiplying fractions in their designated group and continued to work through problem set material adapted from EngageNY. On day five, student work concluded with administration of the posttest, which was identical in format and content to the pretest. Following the posttest, students in Group E (the experimental group) were asked to complete a post-experiment survey where they summarized their attitude toward the use of the SIM (Rectangular Squares) and articulated how they felt it affected their learning.

After reviewing the survey forms, the researcher returned to the school for a sixth day of data collection. On March 16, 2015, she conducted a small focus group interview with the Group E participants during morning work time. Students were interviewed in the library for additional insight and clarification of survey findings. Interview questions were determined upon review of survey responses. The interview session was 20 minutes in length and the audio recorded in MP3 format on the researcher's iPad.

Data Collection and Analysis

Prior to the start of the study (one week before administration of the survey and pretest), consent forms were administered to all potential student participants. The researcher explained and clarified the purpose of her study for student understanding. Students choose whether they wished to participate in the study, and all forms were collected. All 24 students gave consent to participate in the study. Students also received consent forms to take home to give to their parent/guardian. The consent form outlined the purpose of the researcher's study, and highlighted any potential risk factors (for this study, there were no apparent risk factors). Given the Hispanic population and a large percentage of Spanish speaking family members, the consent form was translated by school personnel and sent home in Spanish where applicable. Consent forms were to be returned to school by February 27, 2015.

Parents were encouraged to contact the researcher with questions and concerns before returning the form to school, signed and dated, having indicated whether they give their child permission to participate in the study. Of the 24 students, twelve parents returned permission slips granting consent to participate in the study. Students whose parents had not given consent to their child to participate still completed activities and lessons during data collection; however data from these students were not collected for the findings report. The twelve participating students were equally distributed in both grouping.

Data was collected during the first two weeks of March in 2015 (March 9nd-March 16th). To clarify, the data collection schedule was as follows:

- Day One (March 9th): Pretest
- Day Two (March 10th): Problem Set 1
- Day Three (March 11th): Problem Set 2
- Day Four (March 12th): Problem Set 3
- Day Five (March 13th): Posttest, Student Survey
- Day Six (March 16th): Focus Group Interviews (Group E Participants)

There were several instruments used in conducting this research. Data was collected through administration and collection of the problem set fraction activity sheets and the pretest/posttest. To strengthen instrument validity, the same problem set activity sheets were used with both the control and experimental group. The activity sheets focused on the skill of multiplying basic fractions and consisted of a series of problems aligned with the Common Core within the EngageNY modules. The pretest and posttest were identical documents consisting of 10 Common-Core aligned questions about fractions.

Problem sets were collected and looked at to gauge student progress and make informal observations on student growth. The administered tests were scored for number of correct responses and these results were statistically analyzed using MiniTab and Microsoft Excel software. From this data, the difference of the means in achievement between the control and experimental groups was comparatively analyzed to assess student achievement in the area of multiplying fractions. All questionnaire and interview responses from students were reviewed and the results included in the findings portion of the study.

To support anonymity and protect the identities of participants, student identities were coded with a numerical prefix (1-6) and either a C or E, indicative of the student group (control or experimental). All work samples from students who were not participating in the study due to parental request were shredded through a high security paper shredder at the conclusion of the data collection; no findings were interpreted nor released. All student work samples were scanned into personal computer files on the researcher's computer, and the physical work samples were shredded upon publication of the research study. All findings entered digitally into software programs and stored as files on the researcher's computer were password protected.

Validity and Reliability

External variables were minimized through careful consideration of the participants and the setting. There was a threat to internal validity with this study due to the testing design. The pretest and posttest were identical in content; hence, students may have done better on the posttest due to being pre-exposed to the test material on the pretest. Several extraneous variables were considered, including prior exposure to the material and student effort. The researcher attempted to make the groups as homogeneous as possible to minimize these variables.

The instruments chosen for data collection were reliable in their consistency of repeated use of the same assessment material. By nature of being a new SIM, the Rectangular Fraction Model lacks reliability as it was not clinically tested prior to implementation. The data collected was rich and high quality through its authenticity. The study design's reliability was further strengthened through use of New York State's curriculum aligned with the Common-Core learning standards and EngageNY.

Results

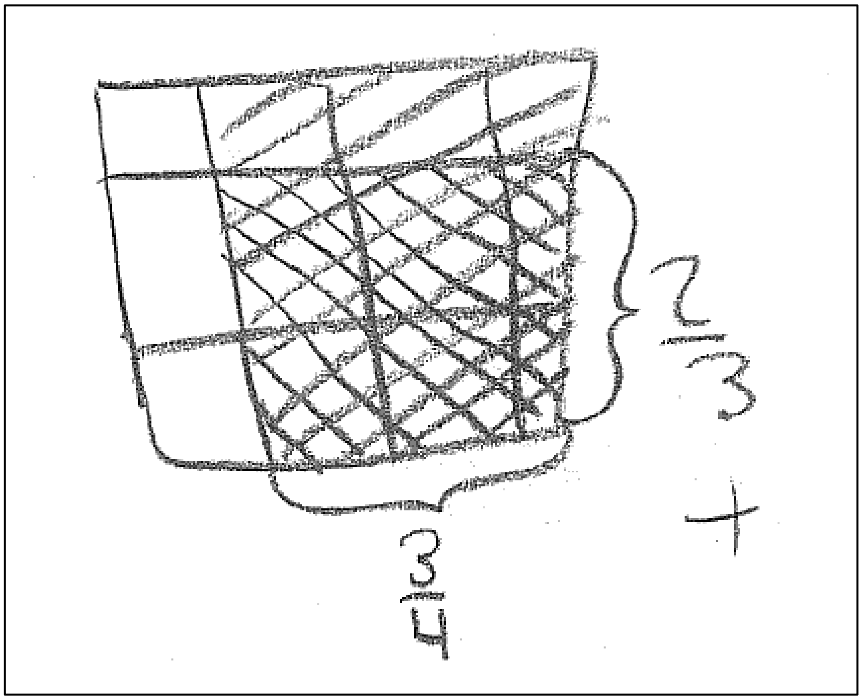
Assessment Data

The focus of this study was exploring the effectiveness of using a simple improvised manipulative (SIM), the Rectangular Fraction Model, to further conceptual understanding of multiplying fractions in a fifth grade classroom. On both the pre-test and the post-test, the directions asked students to solve each problem by drawing a rectangular fraction model to explain their thinking, and then to write a multiplication sentence. These explicit directions were taken directly from a Module 4 EngageNY assessment and serve as grade level expectations in the fifth grade Common Core curriculum. Since the research goal was to evaluate the effectiveness of use of a SIM in helping students draw accurate rectangular fraction models, the researcher decided to focus *only* on the drawing of the rectangular fraction model.

For the purpose of scoring the assessments, the researcher gave one point credit for an accurately drawn rectangular fraction model. To get full credit for any drawing, the student must show the rectangle correctly partitioned by the appropriate unit fractions and the correct fractional amount (part of the whole) shaded. Labels were preferred but not necessary to receive full credit. Most students were able to accurately write a multiplication sentence illustrated by the rectangular fraction model. As there were five items on both the pre and post assessments, student scores could range from 0 to 5. All assessment data collected from this study was coded and organized into informational tables.

To illustrate a full credit rectangular fraction model, enclosed is a snapshot (Illustration 1) of a student's correctly diagramed model. For correctly partitioning and marking the diagram, the student received full credit.

Illustration 1: Student illustration of a rectangular fraction model.



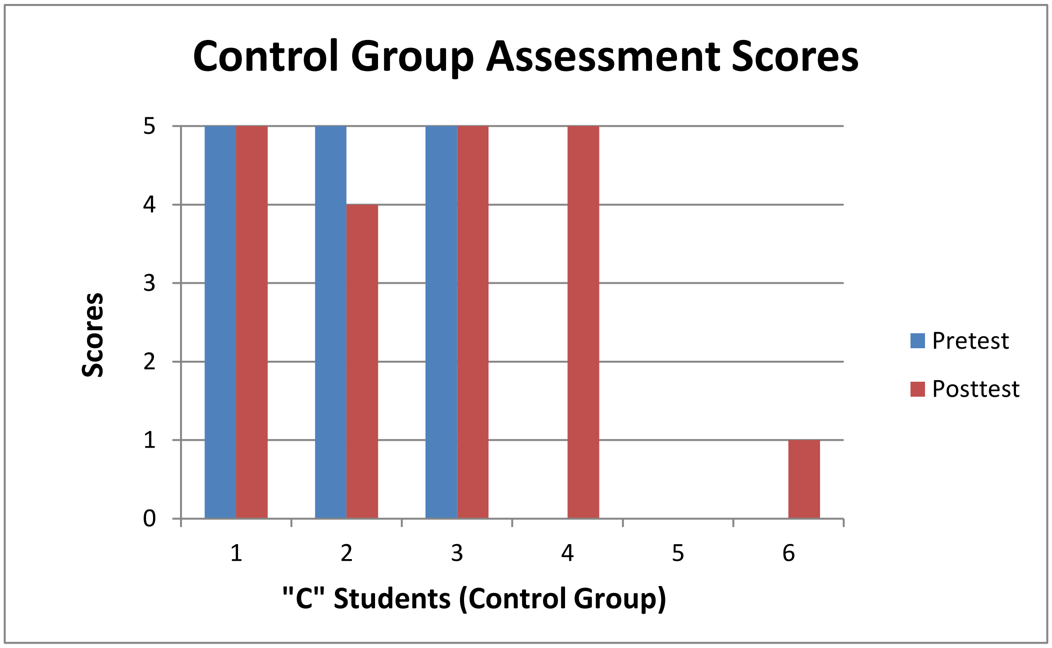
In the above example (Illustration 1), the student received the full point of credit. The stated multiplication problem was $\frac{2}{3} x \frac{3}{4}$ and the student needed to draw the diagram representation. First, this student correctly partitioned the rectangle horizontally (partitioned into thirds) and also correctly partitioned the rectangle vertically (partitioned into fourths). For the second step, the student shaded the appropriate fractional amounts (two of the thirds are shaded using diagonal crosshatchings, and three of the fourths are marked using contrasting diagonal crosshatchings). This student included labels for both fraction factors, an additional step that further exhibited her understanding of this material.

All pretest and posttest assessments were scored by the researcher and the results documented for each student in both the control and experimental groups. The resulting scores of the pre-test assessment and post-test assessment for research participants are summarized in the tables below (Table 1). This data is visually extrapolated and presented in Graphs 1 and 2.

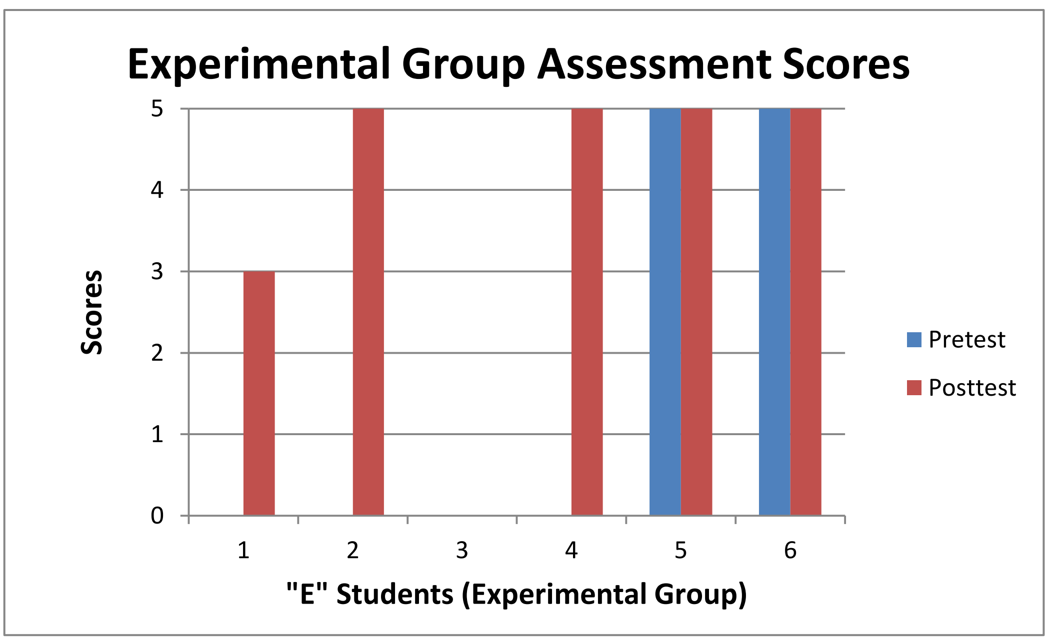
Table 1: Individual Findings on Assessments

Student nC	Pretest $T1$	Posttest $T2$	Achievement $T2 - T1$
Control Group			
1C	5	5	0
2C	5	4	-1
3C	5	5	0
4C	0	5	+5
5C	0	0	0
6C	0	1	+1
Control Average	$\mathbf{2.5}$	$\mathbf{3.\overline{3}}$	$\mathbf{0.8\overline{3}}$
Experimental Group			
1E	0	3	+3
2E	0	5	+5
3E	0	0	0
4E	0	5	+5
5E	5	5	0
6E	5	5	0
Experiment Average	$\mathbf{1.\overline{6}}$	$\mathbf{3.8\overline{3}}$	$\mathbf{2.1\overline{6}}$

Graph 1: Individual Findings on Assessments



Graph 2: Individual Findings on Assessments (Experimental Group)



In the Table 1, data is recorded as collected by the researcher on both the pretest and posttest dates. Student names were coded with a numerical prefix (1-6) and either a C or E, indicative of the student group (control or experimental). Pretest (T1) and posttest scores (T2) are recorded as scored from the corresponding assessment. Students were given one point for every correctly drawn rectangular fraction model on their paper.

Looking at the pretest and posttest score data and comparing results, it can be observed how much progress, or lack thereof, every student individually made. Student 4C showed a 100% percent increase in achievement, as he scored a 0 on the pretest and a 5 on the posttest. Student 6C revealed an achievement gain of +1 (20%). Three control group students showed no increase in learning (no change in score from pretest to posttest), whilst one student displayed slight regression (20%). This student scored a perfect 5 on the pretest but incorrectly drew a fraction model on the posttest, resulting in a score of 4.

Students in the experimental group also showed mixed achievement, but one more student showed a gain in achievement than in the control group. Students 2E and 4E showed tremendous progress. They both scored 0 on the pretest and scored 5 on the posttest, making a 100% increase in learning. One other student, 1E, showed progress. He scored a 0 on the pretest and scored a 3 on the posttest, demonstrating a 60% increase in learning. Similar to the control group, three students in the experimental group did not show any progress in learning. There were no cases of regression in the experimental group.

Overall student achievement on the two tests was calculated by finding the difference between the pretest and posttest grade ($\mathrm{T2} - \mathrm{T1}$). Achievement scores other than neutral scores of zero were given either a positive or negative marking to denote whether the researcher found there to be student growth or regression. In the control group, one student saw a maximum gain in achievement (five points) and one student saw a minor one point gain in present level of achievement when working with this skill (Table 1). Three students saw no difference in level of achievement, and one student showed minor regression of one point. In the experimental group, two students saw an achievement gain of five points and one student an achievement gain of three points from the baseline. The other three students in the control group did not see a change in present level of achievement. No student in the experimental group saw regression in present level of achievement. From this data, it can be concluded that the experimental group saw greater achievement than the control group.

To better understand the data, the researcher summarized each category of data analyzed in Table 1 and presents overall achievement information in the table below (Table 2). The researcher took both the control and experimental sets of data and averaged the cumulative data through using the formula:

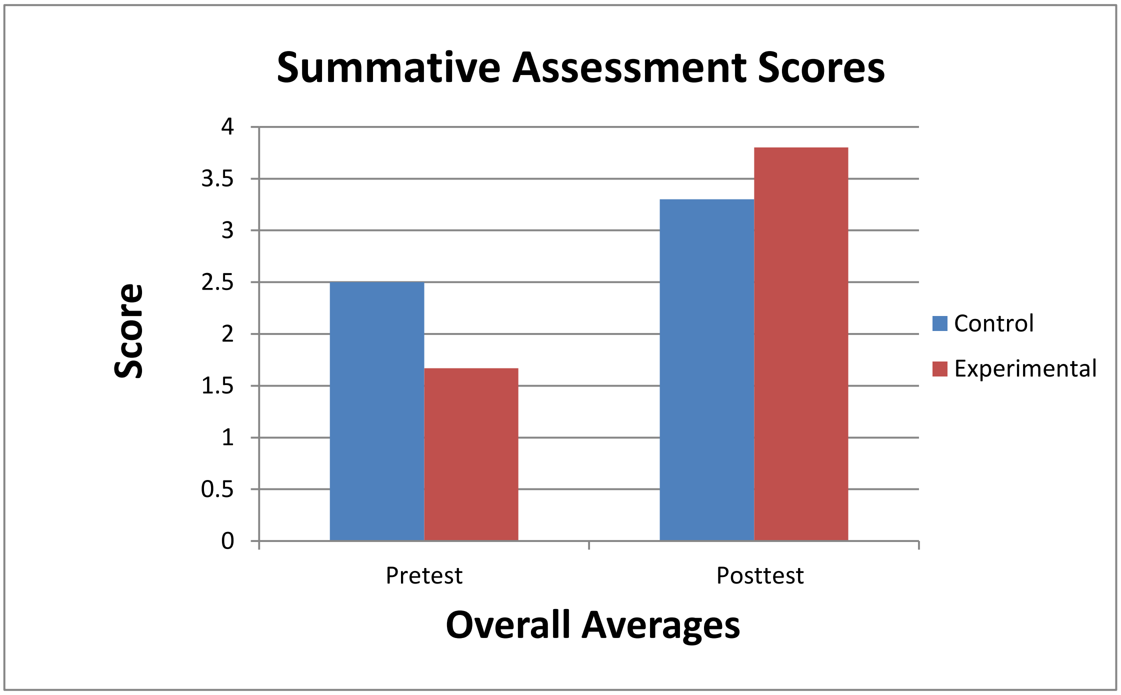
$$average = \frac{sum\ of\ scores}{number\ of\ students} \quad .$$

The items in Table 2 are presented in visual representations (Graphs 3 and 4).

Table 2: Summative Data Report

Group	Pretest Average	Posttest Average	Achievement Average
Control	2.5	$3.\overline{3}$	$0.8\overline{3}$
Experimental	$1.\overline{6}$	$3.8\overline{3}$	$2.1\overline{6}$

Graph 3: Summative Data Report-Assessment Averages



Graph 4: Summative Data Report-Achievement Averages

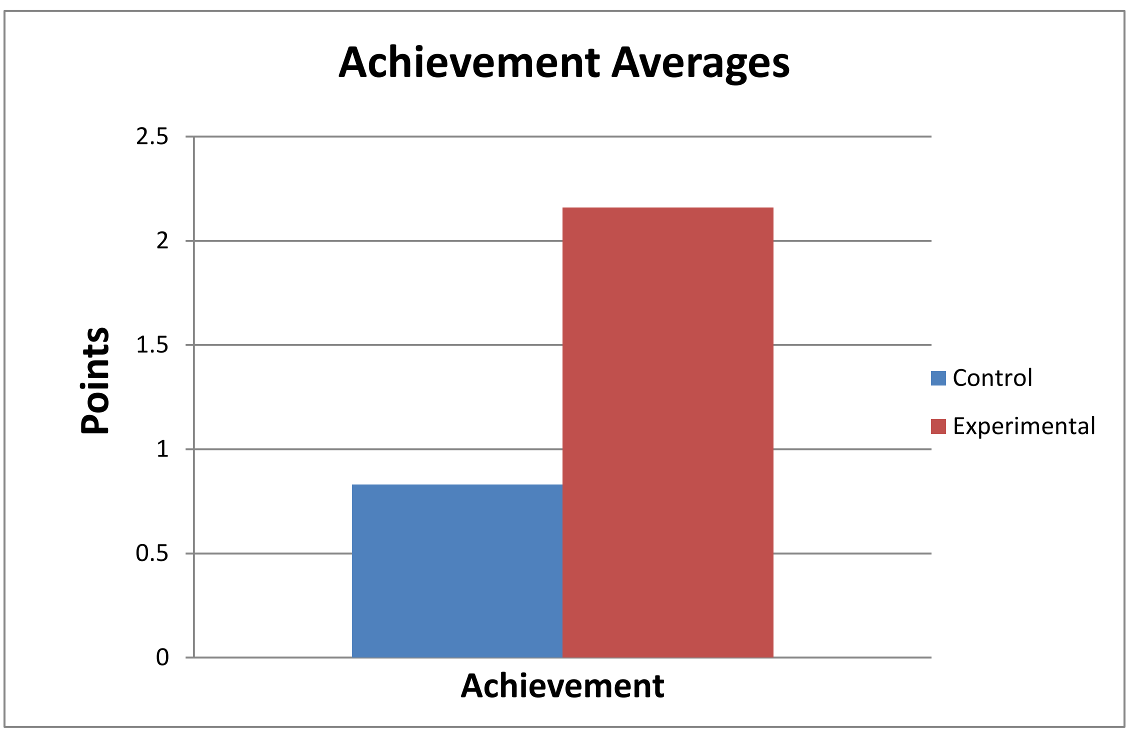


Table 2 presented summative averages of all data collected from both control and experiment participant groups. When averaging all participant pretest scores, the control group scored a 2.5, and the experimental group achieved an average score of $1.\overline{6}$, with the $\overline{6}$ denoting a repeating decimal value. Here, it is evident the control group scored higher, but this changed in the summative posttest score report. When looking at the posttest scores, the control group scored on average a score of $3.\overline{3}$, and the experimental group scored an averaged $3.8\overline{3}$. When looking at the mean scores, we see that the experimental group $(2.1\overline{6})$ scored higher than the control group $(2.08\overline{3})$.

In review of all the data, it would appear that the experimental group performed better on the final assessment, particularly when comparing the final posttest to baseline present level of academic performance data. However, these conclusions are superficial without greater statistical analysis. Using MiniTab statistical analysis software, the teacher calculated whether the difference in achievement between the control and experimental groups was statistically significant. A *p*-value was derived from input data through performing a difference of means test. Looking at the difference in the average achievement rates, the researcher found that the intervention was not statistically significant, given a *p*-value of p=0.17. Although students in the experimental group scored better on the posttest and appeared to have a better understanding of the concept taught, the difference between the control and the experimental group was not statistically significant. This implied that the use of the SIM was not more effective than the traditional teaching approach.

Questionnaire Data

Achievement level was one area of interest for the researcher in studying the impact of a self-created SIM on student learning. This was addressed through administration of both the pretest and posttest assessments. A second area of interest for the researcher was student understanding of material; she asked: How can student use of the Rectangular Fraction Model influence student conceptual understanding of abstract topics? To address this research question, a questionnaire was created and given to experimental participants, and the information gathered from student insight is summarized below. Questions 1 and 3 are listed below with the number of responding participants (Table 3 and 4).

Table 3: Question 1-On a scale of 1-5, how much did you enjoy using the rectangular Fraction Model?

1 (I did not enjoy using the tool)	2	3	4	5 (I greatly enjoyed using the tool)
0	0	1	2	3

Looking at the data, it is clear that students held different opinions regarding the Rectangular Fraction Model. Three of the students greatly enjoyed using the Rectangular Fraction Model SIM to multiply fractions. Two of the students in the experiment group indicated that they enjoyed using the Rectangular Fraction Model a bit, and one student felt neutral. No student in the experimental group indicated dislike of use of the SIM.

Table 4: Question 3- Which was more helpful for multiplying fractions-drawing a diagram on the problem set or using the rectangular fraction model?

Drawing	Rectangular Fraction Model
4	2

Question 3 from the questionnaire informed the researcher that when given the option to draw a rectangular fraction model or use the SIM while working through problem set and assessment materials, students preferred using a traditional drawing to help visually illustrate the process of multiplying two fractions. Four students preferred a traditional drawing, while two preferred use of the Rectangular Fraction Model SIM.

The second question on the questionnaire asked for student input as to whether use of the Rectangular Fraction Model SIM helped students multiply fractions. Of the respondents, two students indicated that they did not find the rectangular fraction model helpful. These students cited having already known the content well enough that implementation of a manipulative was not beneficial. Four students gave a response indicating they found the SIM helpful. The students were asked to elaborate on their response. Below are the full responses (Table 5).

Table 5: Student Responses to Questionnaire

Student	Did the Rectangular Fraction Model help you multiply fractions? (Yes or No)	Explain your Response
1E	Yes	"The Rectangular Fraction Model helped me do it right."
2E	No	"It did not because you can multiply fractions without it."
3E	Yes	"Yes because it is a visual of what I am doing."
4E	Yes	"It helped me multiply fractions because some were hard."
5E	Yes	"It was easy to see the fractions."
6E	No	"It didn't really help me because I already know how to multiply fractions."

Focus Interview

On the Monday following the week of data collection, the researcher returned to the research site and met with the experimental group in the school library. The researcher collected richer descriptive data on the topic of research and overall effectiveness of implementation of the Rectangular Fraction Model SIM through conducting a focus interview with the experimental group. Prior to her visit, the researcher reviewed the data collected through survey responses and scored the assessment pieces, and through her analysis she compiled a list of several interview questions. The researcher sat in the middle of the table with the six students seated around her.

The conversation was recorded to promote accurate transcription and field notes taken. This data was coded and themes were acknowledged (Table 6).

Table 6: Themes from Focus Interview with Experimental Group

Conversation with Experimental Group (Coded)

Question 1: What did you like best about using the Rectangular Fraction Model?

Themes:
A: Interactive
B: Color
C: Different Learning Style
D: Visual
E. Ease of Use

Student	Code	Response
1E	E	It was easier to use the model.
2E	A, D	The two pieces of plastic matched…It was like magic!
3E	C	It was a nice visual.
4E	A,B, D	It helped me see how to multiply two fractions together. I liked the different colored markers.
5E	B, C, D	Using different colored markers helped me visualize. It would have been nice if our teacher used these.
6E	A, D	It was easy to see one fraction on top of another. I liked how they were see-through.

Conversation with Experimental Group Continued (Coded)		
Question 2: Did using the Rectangular Fraction Model help you multiply fractions? **Themes:** **A: Easier** **B: Prefer Drawings** **C:Knew the Material** **D: Timing** **E: Unsure**		
Student	**Code**	**Response**
1E	A	It was easier to use the model.
2E	B, D	I liked just doing the drawing. It was hard to do both the problem set and use the RFM.
3E	A	I liked seeing the two fractions.
4E	D	I had a hard time completing the problem set because I was busy using the RFM.
5E	C	I'm good at math and I knew how to do this.
6E	C, D	It might help if we learn to do this before learning to do the drawing.

It can be concluded from interview responses (Table 6) that students in the experimental group liked using the Rectangular Fraction Model. Many found the new manipulative exciting and novel. Others greatly appreciated the visual nature of the SIM and thought the design of the manipulative helped them understand the concept of using a rectangular array to multiply fractions. Color (through use of colored markers) was useful in helping students see the two fraction factors in each multiplication sentence. Although all six students were in agreement that the learning tool could certainly benefit students learning to multiply, several did not feel the SIM was personally helpful as they already understood the concepts and skills presented.

Discussion and Conclusion

In summary, this study tended to answer the following questions:

- How does implementation of a SIM, the Rectangular Fraction Model, impact 5th grade students' achievement in a mathematics class at an urban Chautauqua County elementary school in Western New York?
- How does use of a concrete representation affect students' conceptual understanding of abstract material as taught through the Common-Core aligned EngageNY curriculum?

Students in the experimental group were given a math manipulative, the Rectangular Fraction Model, and were given three days to explore the simple improvised manipulative and use it to help them complete problem set materials. The hypothesis was that student achievement will be higher with manipulative use as an intervention, and that conceptual understanding will be heightened through use of the concrete learning tools. The experimental group's assessment data and student opinions confirmed the hypothesis. However, the difference in mean assessment results between the control and experimental group does not show to be statistically significant, and thus the use of the SIM is not more effective than the traditional methods.

Limitations

Although it seems that students in the experimental group achieved higher score on posttests and had better conceptual understanding of the abstract concept, there are several limitations of the study. This study looked at a relatively small sample population of fifth grade students; only twelve students were included in the study from a class of 24 students. These twelve students from a unique population represent a statistically insignificant population of fifth graders in Western New York; hence no generalizations can be made regarding the useful of this tool with fifth graders across the region.

This small sample size affected the validity of the statistical analysis. The fewer the number of participants, the less strong the data from resulting statistical analysis; comparing six outcomes to six outcomes does not produce a *p*-value of precision. In further regard to the analysis of the data, the scoring system used to assess student learning on the formal assessment pieces (one point for every correct response) swayed the derivation of the p-value score. Scoring using this point system, with scores in the 0-5 range, lessened the precision of the p-value.

Another area of limitation would be the data collection dates. The researcher needed to amend the time frame of her study due to logistical concerns with the school. Hence, research data was completed later than initially planned. This gave the student sample population additional time to work on the skills presented in this study in their nature lecture-style classroom environment. In both the survey results and focus interview, several students from the experimental group expressed having already been exposed to this material; therefore they did not find the implementation of a learning tool particularly beneficial in strengthening conceptual understanding of the items presented.

In this study, the manipulative implemented as an intervention was researcher-created. The SIM created, the Rectangular Fraction Model, has no data on its reliability as a manipulative designed to differentiate instruction. Although the of the SIM in this study provided the researcher with an opportunity to test the manipulative's effectiveness in the natural classroom setting, the lack of proven reliability served as a limitation.

Lastly, the researcher's inability to be with both the control and experimental groups serves as a limitation. The assignment of the experimental group to work in the cafeteria prevented the researcher from observing both groups and their work. The cafeteria environment (the only available location for the research) was at times loud with cleaning staff and often students were easily distracted by passing students or the cafeteria staff. Although the classroom teacher was given directives to follow and she gave a report of student behavior, the researcher cannot be certain the control group was treated with its prescribed expectations.

Applications for Practice

Objectively, this experiment did not show use of the Rectangular Fraction Model to be more effective than the traditional teaching (drawing rectangular arrays) with student learners as shown in analysis of the assessment data. Subjectively, it could be noted that the experimental manipulative could have some level of success with struggling learners, as supported by student response on questionnaire material and through the focus interview.

Students in the experimental group supported the use of colored markers while using the Rectangular Fraction Model, and found the inclusion of color beneficial in visualizing the two unit fractions being multiplied. It can also be noted that while some students greatly enjoyed the visual nature of the manipulative; others felt the presentation and use of the tool to be too remedial. This stresses the need for differentiating instruction to meet individual learner needs.

In consideration of the study’s outcomes, the researcher supports use of the Rectangular Fraction Model as an introductory activity aimed at introducing the topic of unit fraction multiplication through pictorial representation. It would serve beneficial to students just exploring multiplication of two fractions, and would be a great visual aid. She would not advise continued use of the RFM through the unit, unless students are still struggling with the concept of drawing the fraction model.

As explored in the review of the literature, concrete manipulatives have shown to benefit a wide variety of learners (Puchner et al., 2008, Sherman & Bisanz, 2009). To maximize potential learner outcome and heighten student achievement, teachers should explore available manipulatives in their school setting and research the reliability and validity of using any learning tool before deciding to implement the tool into a lesson.

Implications for Further Research

Further research can be done on the effectiveness of SIM use in the teaching mathematics. SIMs are cost effective and easy for the teacher to create using local materials (Andrews, 2004). Although the study did not prove the Rectangular Fraction Model to be more effective than a traditional teaching approach, a similar SIM concept could prove more successful in meeting student needs. Future research could be done on other means of helping students learn to multiply unit fractions using concrete learning tools. It is suggested that future research is done exploring the effects of the SIM on students in an AIS setting.

This study focused on exploring the impact of a concrete manipulative in the mathematics setting, but great research has been done in the field of virtual manipulatives and their impact on mathematics education (Ganesh & Middleton, 2006; Widmer & Sheffield, 1994). As an avid technology user and firm believer in the power of instruction technologies, the researcher advocates that further research is completed in this area of study to explore the impact of virtual manipulatives on student learning outcomes. For instance, it would be interesting to explore the impact of drawing rectangular fraction models on a SMART Board.

Through exploration of fifth grade EngageNY lesson materials and extrapolation of a concept presented in the modules, the research connected this study to the Common Core State Learning Standards. Through initial reviews of the available research, there are few studies to look at teaching to these standards using manipulatives. The researcher recommends that greater research be done to lessen this deficit.

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Appendices

Appendix A: Experimental Group Survey

Please answer the following questions. Your opinion is important!

1. On a scale of 1-5, how much did you enjoy using the Rectangular Fraction Model?

1 (I did not like using the tool)   2   3   4   5 (I really liked using the tool)

2. Did the Rectangular Fraction Model help you multiply fractions? Why or why not? Explain your answer.

____
____
____
____
____

3. Which was more helpful for multiplying fractions-drawing a diagram on the problem set or using the Rectangular Area Model?

Drawing   Rectangular Fraction Model

4. What was the best part of participating in this study?

____
____
____
____

Appendix B: Pretest/Posttest Assessment Instrument

Name________________________________ Pretest

1. Solve. Draw a rectangular fraction model to explain your thinking. Then write a multiplication sentence.

a. $\frac{2}{3}$ of $\frac{3}{4}$

b. $\frac{2}{5}$ x $\frac{3}{5}$

c. $\frac{2}{7}$ of $\frac{4}{5}$

d. $\frac{3}{9}$ x $\frac{2}{5}$

2. Noah mows $\frac{2}{5}$ of his property and leaves the rest wild. He decides to use $\frac{3}{7}$ of the wild area for a vegetable garden. What fraction of the property is used for the garden? Draw a picture to explain your answer.

Appendix C: Problem Sets

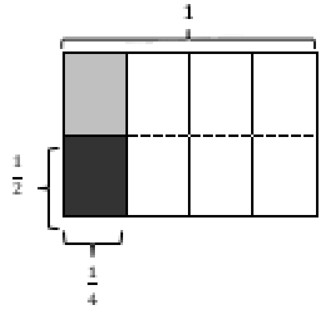
Problem Set 1

Name ______________________ Date ____________

1. Solve. Draw a rectangular fraction model to show your thinking. Then, write a multiplication sentence. The first one has been done for you.

a. Half of $\frac{1}{4}$ pan of brownies = $\frac{1}{8}$ pan of brownies

$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$



b. Half of $\frac{1}{3}$ pan of brownies = _____ pan of brownies

c. A fourth of $\frac{1}{3}$ pan of brownies = _____ pan of brownies

d. $\frac{1}{4}$ of $\frac{1}{4}$

e. $\frac{1}{2}$ of $\frac{1}{6}$

Problem Set 2

2. Draw rectangular fraction models of $3 \times \frac{1}{4}$ and $\frac{1}{3} \times \frac{1}{4}$. Compare multiplying a number by 3 and by 1 third.

3. $\frac{1}{2}$ of Ila's workspace is covered in paper. $\frac{1}{3}$ of the paper is covered in yellow sticky notes. What fraction of Ila's workspace is covered in yellow sticky notes? Draw a picture to support your answer.

4. A marching band is rehearsing in rectangular formation. $\frac{1}{5}$ of the marching band members play percussion instruments. $\frac{1}{2}$ of the percussionists play the snare drum. What fraction of all the band members play the snare drum?

5. Marie is designing a bedspread for her grandson's new bedroom. $\frac{2}{3}$ of the bedspread is covered in race cars and the rest is striped. $\frac{1}{4}$ of the stripes are red. What fraction of the bedspread is covered in red stripes?

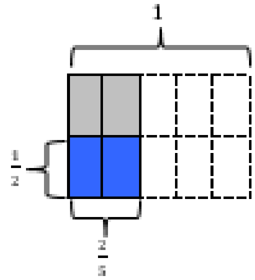
Problem Set 3

Name ______________________________ Date ________________

1. Solve. Draw a rectangular fraction model to explain your thinking. Then, write a number sentence. An example has been done for you.

Example:

$\frac{1}{2}$ of $\frac{2}{5} = \frac{1}{2}$ of 2 fifths = 1 fifth



$\frac{1}{2} \times \frac{2}{5} = \frac{2}{10} = \frac{1}{5}$

a. $\frac{1}{3}$ of $\frac{3}{4} = \frac{1}{3}$ of ____ fourths = ____ fourth

b. $\frac{1}{2}$ of $\frac{4}{5} = \frac{1}{2}$ of ____ fifths = ____ fifths

c. $\frac{1}{2}$ of $\frac{2}{2} =$

d. $\frac{2}{3}$ of $\frac{1}{2} =$

e. $\frac{1}{2} \times \frac{3}{5} =$

f. $\frac{2}{3} \times \frac{1}{4} =$

Appendix D: Consent Forms

Dear Parent or Legal Guardian:

My name is Rachael Kibler and I am a certified New York State teacher and graduate student researcher in the Curriculum and Instruction Program at SUNY Fredonia. For five days in March (March 9th-13th), I will be working with (Classroom Teacher's) students to conduct research exploring new, hands-on ways of teaching fifth grade math.

The purpose of this research study is to explore the impact of learning tools on student achievement in mathematics. I will be working with the students as they use a hands-on learning tool, the Rectangular Fraction Model, to practice multiplying fractions. The learning tool I am introducing to students supports the Common-Core curriculum and EngageNY materials used in class.

All fifth grade students at (Elementary School) will participate in hands-on exploration of multiplying fractions during math center time. Throughout the research, students will complete EngageNY problem sets while working with the tools. A pre- and post-test assessment will be given to all students and assessed to determine if the use of the Rectangular Fraction Model was beneficial. Additionally, students will complete a survey on both prior experiences working with hands-on learning tools and their overall satisfaction with using the learning tool.

For my research, I will be collecting work samples to document as student data. This data will be analyzed to determine the effectiveness of hands-on learning tools. The findings of this study will help guide teachers in developing new ways of teaching mathematics in the fifth grade. The findings will also reflect student opinion on use of new learning tools.

All student work collected and analyzed as part of my research will remain confidential and anonymous. No reports about this study will contain your child's name. There are no physical or academic risks involved with your child's participation in this study.

Participation in this study is entirely voluntary. Your child will not be penalized in any way. Please select whether or not you would like your child to participate in this study, and sign and return the attached form to school by **Friday, February 27th**. If you have any questions or concerns regarding this research study and your child's participation, please contact Dr. Guangyu Tan or myself.

Thank you,

Rachael Kibler

Guangyu Tan, Ph.D.
Assistant Professor, College of Education
SUNY Fredonia

Rachael Kibler
Graduate Student/Researcher
SUNY Fredonia

Parent/Guardian:

Please select whether or not you would like your child to participate in this study, and sign and return this form to school by **Friday, February 27th**. Thanks!

____

(name of child)

______**YES,** I want my child to be included in the research study on use of learning tools in mathematics.

______**NO,** I do not want my child to be included in the research study on use of learning tools in mathematics.

____________________________________ ____________________

Parent/Guardian Signature *Date*

Dear Student:

My name is Miss Kibler and I am a teacher and graduate student researcher in the Curriculum and Instruction Program at SUNY Fredonia. For five days in March (March 9^{th}-13^{th}), I will be working with (Classroom Teacher) in your classroom to research new, hands-on ways of teaching fifth grade math.

The purpose of this research study is to explore the impact of learning tools on student achievement in mathematics. (Classroom Teacher) and I will work with you during math center time to practice multiplying fractions. Half of you will work with me in the cafeteria, and the other half of you will stay in the classroom with (Classroom Teacher). One group will be the experimental group, which means you get to try a new learning tool!

All fifth graders at (Elementary School) will participate in math center time. Throughout the research week, you will complete problem sets while working with a teacher. You will take a pre- and post-test to see if we have improved at multiplying fractions. You will also complete a survey where YOU get to give your opinion on being an active participant in my research!

Good researchers collect data from their research to determine whether a study was effective or not. As a researcher (and teacher who wants to help you improve in math) I will be collecting data in the form of work samples. What does this mean? It means that I will collect everyone's problem set, survey sheets and tests for scoring. I will then use this data to determine how fifth graders at (Elementary School) learn best.

All work collected and analyzed as part of my research will remain confidential. No reports about this study will contain your name. There are no physical or academic risks involved with your participation in this study.

Your participation in this study is entirely voluntary. You may choose not to participate. There are no penalties for not participating.

Thank you,

Miss Kibler

Please circle your response:

Yes, I would like to participate in this study.

No, I would not like to participate in this study.

__________________________________ ____________________

Student Signature *Date*

Appendix E: IRB Human Subjects Project Approval

FREDONIA
STATE UNIVERSITY OF NEW YORK

3 March 2015

Rachael Kibler
c/o Guangyu Tan, Ph.D.
Curriculum and Instruction
College of Education
The State University of New York at Fredonia

Re: Rachael Kibler—Rectangular Fraction models: An Exploration of the Impact of Concrete Mathematics Manipulative Use in the Fifth Grade

Your research project using human subjects has been determined Category 1, Exempt, under the United States Department of Health and Human Services Code of Federal Regulations Title 45 Public Welfare, Part 46 Protection of Human Subjects, 46.101, Subpart A (b) (1) and/or (2). This document is your approval and your study titled "Rectangular Fraction models: An Exploration of the Impact of Concrete Mathematics Manipulative Use in the Fifth Grade" may proceed as described, beginning on **March 3rd, 2015 and ending on April 30, 2015.**

Thank you for keeping the high standards relating to research and the protection of human subjects under the auspices of the State University of New York at Fredonia.

Sincerely,

Judith M. Horowitz, Ph.D.
Associate Provost, Graduate Studies, Sponsored Programs
and Faculty Development
Human Subjects Administrator