State University of New York at Fredonia
Department of Mathematical Sciences

CERTIFICATION OF PROJECT WORK

We, the undersigned, certify that this project entitled A Study of Students’ Estimation Skills With Shaded Areas by Kaitlyn E. Whitney, Candidate for the degree of Master of Science in Education, Mathematics Education (7-12), is acceptable in form and content and demonstrates a satisfactory knowledge of the field covered by the project.

Teodora B. Cox, Ph.D.
Master’s Project Advisor
Department of Mathematical Sciences

July 21, 2014
Date

II. Joseph Straight, Ph.D.
Department Chair
Department of Mathematical Sciences

7/21/14
Date

Dr. Teresa Brown, Ph.D.
Provost and Vice President for Academic Affairs
At SUNY Fredonia

Date
A STUDY OF STUDENTS’ ESTIMATION SKILLS WITH SHADED AREAS

by

Kaitlyn E. Whitney

A Master’s Project
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Abstract

This study examined high school and college students’ skills to estimate area. During this study, students completed a ten-problem assessment which contained ten different geometric figures that were partially shaded in. Students were instructed to estimate what percentage of the area of the shape was shaded and to explain how they made their predictions for three problems. Each problem was scored on how far off their estimation was from the actual percentage. After completing the assessment, students were also asked to complete a five-question survey. The results of the study indicate that students are much more likely to overestimate than underestimate. Additional results revealed that problems that had a slanted shade line and problems with multiple pieces shaded were the most difficult for students. Other findings showed that there was no significant difference based on gender, but there was a significant difference based on what course students were enrolled in.
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Introduction

This study investigates students’ skills and understanding of area estimation of two-dimensional figures. Specifically, it explores the accuracy and inaccuracy of estimates of the percent area that is shaded in such a shape. This research also examines students’ thought processes while they are forming their estimates. Students experience different thought processes, but their answers will ultimately originate from their true understanding of estimation, percentages, and area. It was expected for students to have a difficult time with this task because these three topics are not generally taught to students at the same time.

My interest in this topic stems from my experiences in the classroom with both estimation and the concept of area. Throughout my field placements I have asked students of many ages to estimate answers throughout many different topics; however, I always have the same problem with this assignment. Students do not want to answer questions unless they know their answer is “correct”, which contradicts the idea of an estimate. It was important to find a topic where students could not compute quickly by hand or with a calculator. The concept of area also presents a struggle for many students. There is not a sole formula to memorize that will work for all shapes, and area is often confused with perimeter.

I wanted to research student estimation skills with area because I believed that combining these two topics would challenge students to force themselves to make an educated estimate instead of mathematically calculating the answer. It is important for students to understand that an estimate is allowed to be incorrect as long as they are applying correct techniques to make the estimate. This study also aims to investigate the steps that each student takes to make their estimation in order to determine which topic students struggle with more.
It is hypothesized that students will overestimate more frequently than underestimate when asked to estimate the percentage of area of a partially shaded shape. Furthermore, it is hypothesized that students of all ages will inordinately estimate quartiles (25%, 50%, 75%). In addition, gender and course will likely have no statistically significant effect on estimation accuracy.

This hypothesis was tested by administering a test to high school and college students. The assessment included different figures which had a percentage of the area shaded in. It was scored by assigning one point for each single percent by which the estimate was incorrect. Thus the lower the score, the better the estimation skills. The goal of the assessment was to obtain the lowest score possible. Figure 1 shows two examples of the shaded area problems from the assessment. The students were instructed to give their estimation of the percent of the area that was shaded.

**Figure 1. Example Shaded Area Problems.**

In addition, students completed a survey after taking the assessment in which they shared information about techniques they used to make the estimates. The following literature review examines the traditional and nontraditional methods of teaching and calculating area and the standards and assessments that are required to be met by the Department of Education at the national and state levels. It also investigates percentage and estimation misconceptions that
students have. Finally, it explores area misconceptions that teachers and students have as well as common misconceptions between area and perimeter that are held by both teachers and students.

**Literature Review**

The purpose of this literature review is to examine prior research pertaining to percentages, estimation, and area. It begins by focusing on traditional and non-traditional attempts at teaching and calculating area for students of all ages. The following research also includes an inspection of the standards and assessments that are put in place and the differences between the old and the new standards. The literature review continues by concentrating on the misconceptions that individuals, both students and teachers, have about percentages, estimation, and area.

**Approaches to Teaching/Calculating Area**

**Traditional methods.** In a classroom, there are many different methods which could be employed to teach students the concepts of area, as well as how to calculate it. Figure 2 shows a traditional method that can be found in various textbooks. Throughout multiple textbooks it can be noted that there are a variety of definitions and problems that can be used to teach the concept of area (Bezuk, 2007).

![Figure 2: Sample Traditional Method Book Lesson on Area and Perimeter from Boswell, Kanold, Larson, & Stiff (2007)]
Cathcart, Pothier, & Vance, 2011; Carter, Cuevas, Day, & Mallory, 2013). Many of these textbooks do not include a detailed, full explanation of what area actually is and how to derive the formula (Boswell, Kanold, Larson, & Stiff, 2007; Charles, Dossey, Leinwand, Seeley, & Vonder Embse, 1998). There are some textbooks which do try to further explain the concept of area (Collins, Dritsas, Frey, Howard, McClain, Molina,…Wilson, 2001; Leschensky, Price, & Rath, 1992).

Other texts provide a definition of area that is followed directly by the formula, which is seen in the middle school text book written by Charles et al. (2001). Immediately thereafter, multiple examples are solved by stating the formula, substituting in for the variables, and solving the equation. There are no examples which challenge the students to derive the formula themselves or think more deeply about what area means. Boswell et al. (2007) offer the same format; however, perimeter is also taught at the same time. The examples throughout the section are mixed between perimeter and area problems which all involve simple substitution. Such a mix could not only cause students to not fully understand area, but also causes confusion between perimeter and area. This confusion could create issues for the remainder of students’ schooling.

Although some textbooks simply state the definition followed by the formula, there are some texts which try to give the students a better understanding of the concept. Carter, Cuevas, Day, and Mallory (2013) provide us two texts which attempt to do this. Both texts allow students to find the area themselves before stating the formula. As seen in Figure 3, in Course 1 Volume 2, there is a shape given in a grid. This figure allows the students to actually see that area is the amount of space a shape covers. The units that the shape covers can physically be counted in order to find the area. At the end of the section in each of the books there are harder
questions which force students to think deeply about area instead of just plugging in numbers to a formula.

**Figure 3.** Sample Traditional Method Book Lesson on Area from Carter, Cuevas, Day, & Mallory (2013).

**Non-traditional methods.** Although the traditional methods that are used in classrooms work for some students, there are other students who need to learn this material in other, non-traditional ways. There are additional methods that can be used to teach mathematics, including the concept of area, that do not include a textbook and a pencil. These ways provide students with a deeper understanding of the concept than a formula does (Lappan & Even, 1989; Marshall, 2006; Nirode, 2011; Zacharos, 2006). One newer method of teaching that has risen in the past decade involves students working with different computer programs. There are various programs that teachers can use which include numerous explanations of area and methods to derive the formula (Bouck & Flanagan, 2009, Dahan, 2011; Naidoo & Naidoo, 2008).

Another non-traditional approach to teaching area involves student use of computers. Jean-Jacques Dahan (2011) suggests that using a Cabri learning environment in schools can vastly increase student understanding of the concept of area as well as the difference between area and perimeter. The Cabri environment involves students working on the computer with a
program that has multiple files used to investigate and come to conclusions about the area of shapes. The activity starts by allowing the students to investigate the area of a disk. This activity begins with just one yellow square on the screen which is labeled with an area of one. Next, a larger pink square appears and the students are asked what number should the yellow square be multiplied by in order to get the pink square. The students would be able to easily answer this question with the correct answer, four, which is the area of the larger square.

Following the completion of this step, a blue disk appears. The new question that is presented to the students asks what number the yellow square has to be multiplied by to get the blue disk. From this question, the students derive the formula for the area of a disk. This program uses the same type of process to find the lateral area of a cylinder and the area of a sphere (using 3D Cabri). The Cabri learning environment also has a file dedicated to aiding students in coming to the conclusion of the relationship between distance enlargement and area enlargement by using a smaller and larger picture of the same figure. An example of the Cabri learning environment is seen in Figure 4.

Dahan believes that using this Cabri environment will allow students to more thoroughly understand area and give them the ability to find the area formulas of different shapes.

Another computer program that can be used to understand area is examined by Emily Bouck and Sara Flanagan (2009). Their article discusses the use of virtual manipulatives in the
Virtual manipulatives are manipulatives (geoboard, Base 10 Blocks, square tiles, etc.) that are on the computer and can be manipulated with a computer mouse. An advantage of using virtual manipulatives is the fact that students can continue to work without the fear of ever running out of stock of their manipulatives. The concepts of area and perimeter are discussed in this article. A great way for students to learn these topics is through the use of manipulatives, in this case Polyominoes, or square tiles on the computer which are seen in Figure 5. The Polyominoes are exceedingly easy to work with because they allow the student to drag them around the screen, add and delete shapes, and even change the color of each tile.

A student can be presented with a figure that has a given perimeter and be asked to use the tiles to find the area, or vice versa. Students are also able to cover a larger shape with the smaller tiles to determine how many tiles it takes to cover the shape. These virtual manipulatives assist students in understanding and finding the area of figures.

A different, non-traditional approach to teaching area is by reading a book and completing a hands on activity. In order to review the concept of area in a fifth grade classroom, Ashley Deters and Tracy Espejo (2011) recommend the short story *Spaghetti and Meatballs for
All!, by Marilyn Burns, as an opener to the topic. In this book, a couple, Mr. and Mrs. Comfort, decided to have a dinner party. Mrs. Comfort made a perfect seating arrangement with all of the tables spread out. As their guests began to arrive at the party, they moved the tables and seats around so they could fit more people at one table. Once everyone had arrived there were not enough seats left for everyone and the seating arrangement had to be put back to the original design. After the story was read the students reviewed all of the seating arrangements and drew them in order to calculate the area of each separate seating arrangement. The teacher also had the students recreate each of the seating arrangements in the classroom with their own desks in order to fully see what was happening to the area and perimeter. The next activity the class did involved bags of topsoil, which were really square tiles, to create gardens. The students were arranged in groups of three and had to construct three gardens, one of which could not be rectangular. They were then asked to trace their garden onto graph paper and label it with the correct perimeter and area. These two activities allowed students to fully understand the concept of area instead of just telling them to plug numbers into a formula. Approaches to teaching, both traditional and non-traditional, should align with the standards put in place by the state and any assessments that the students are expected to take.

**Standards/Assessment**

The New York State and country curriculum has continued to change through the past century and is undergoing a transformation now. Under the Common Core, New York State is shifting around the topics that are taught in different grades. Many topics are introduced to students at an earlier age than ever before (Common Core State Standards Initiative, 2012; New York State Board of Education, 2005). Some of these shifts are causing gaps in student
achievement as the assessments get increasingly harder at a younger age (JMAP, 2013; New York State Testing Program, 2013).

The Mathematics Core Curriculum was developed in 2005 by The New York State Board of Education. Under these standards the concept of area was introduced in fourth grade by having students find the area of a rectangle by counting the number of unit squares the rectangle covers. This topic was introduced under the standard 4.G.4., find the area of a rectangle by counting the number of squares needed to cover the rectangle. This is as far as the curriculum had students go in fourth grade. After this subtle introduction, the Core Curriculum did not have students revisit area again until sixth grade under the standards 6.G.2., determine the area of triangles and quadrilaterals (squares, rectangles, rhombi, and trapezoids) and develop formulas, 6.G.3., use a variety of strategies to find the area of regular and irregular polygons, 6.G.7., determine the area and circumference of a circle, using the appropriate formula, and 6.G.8., calculate the area of a sector of a circle, given the measure of a central angle and the radius of the circle (New York State Board of Education, 2005). The sixth grade standards held students accountable for developing area formulas for triangles and quadrilaterals as well as how to use them. They were also responsible for being able to use different methods to find the area of both regular and irregular polygons. The sixth grade material then had students calculate the area of a circle and a sector of a circle. Aside from calculating areas using only a formula, these students were also given polygons on a coordinate plane and asked to find the area. The last topic the sixth graders were responsible for was estimating the area of different figures. Although the idea of area was introduced in fourth grade, it was not until sixth grade when students fully learned the concept.
New York State has recently adopted the Common Core State Standards, which were finalized in 2012. These standards introduce the topic of area in third grade. Not only are the students responsible for knowing what area is and finding it by counting how many units it takes to cover the figure, but this curriculum goes into depth using formulas. Third grade students are now responsible for finding the area of a plane figure by tiling the entire figure and relating this to the formula. An example of this type of question is shown in Figure 6. The Common Core has students work to find the formula, and then they are asked to implement it. These standards also request third grade students to break rectilinear shapes into non-overlapping rectangles in order to find the area. This curriculum is much different than the New York State Mathematics Core Curriculum.

The New York State Board of Education publishes the yearly tests administered to students. In May of 2010, the grade six mathematics test included a question asking the students to find the area by counting the number of units the figure covered, as seen in Figure 7. Even in sixth grade the students are not always required to use the formula for area but are just tested on the actual meaning of area.
The New York State Testing Program has released Common Core sample questions for each grade level. The first sample question relating to area is in the fourth grade sample booklet and is shown in Figure 8.

**Figure 8. Fourth Grade Common Core Sample Problem from**
New York State Testing Program (2013).

The area of Ken’s rectangular garden is 480 square feet. The garden is 24 feet wide. What is the length of fencing Ken will need to buy in order to fence in the garden completely on all four sides?

**Show your work.**

This question not only requires students to know the formula of the area of a rectangle, but also forces them to solve this formula for a variable. Once the students have used the area formula
they then must use the perimeter formula to figure out the total amount of fencing that is needed. This requirement could cause major issues for students if they do not yet have a concrete understanding of the difference between perimeter and area.

Assessments that are used in the middle schools and high schools are even more demanding. On the JMAP website is compiled every mathematics state exam question that has been used in the past few years. On the website one can also create tests and worksheets from past exam questions for students to use as practice. It includes worksheets and tests for each concept as well as assessments with mixed topics. There are plenty of worksheets for algebra and geometry students to use that involve area problems. These problems range from ‘easy’ to ‘hard’. One type of problem that arises in high school is finding the area of a shaded region. This type of question is asked on state tests as well as on the SAT (Dome Exam Prep, 2013).

JMAP has shaded region questions for students to practice. Figure 9 shows a shaded region problem that uses concrete numbers. In this problem, the students can distinctly find the area of each rectangle and can do the numerical mathematics to find the area of just the shaded region. Figure 10 uses variables instead of actual numbers. This problem requires students to calculate the area of each square in terms of the variables $x$ and $y$, and then try to solve for just the area of the shaded region. Such a problem forces the students to
manipulate variables, which can be challenging at times. Both types of questions can be seen on state testing at the high school level.

**Figure 10. Shaded Region With Variables from JMAP (2013).**

The accompanying diagram shows a square with side $y$ inside a square with side $x$.

![Shaded Region Diagram](image)

Which expression represents the area of the shaded region?

1) $x^2$  
2) $y^2$  
3) $y^2 - x^2$  
4) $x^2 - y^2$

Area is often a topic that students struggle in and it is made even more difficult when paired with percentages.

**Percentage Misconceptions**

Percentages are used throughout schooling and in many jobs. Various misconceptions about percentages hold students and adults back from excelling in their everyday lives. There are common mistakes that people make when calculating percentages, but there are also countless individuals who do not understand percentages and how to calculate them. Problems in school arise that students are unable to figure out because they were never taught a lesson on how percents are used (America’s Choice Mathematics Navigator, 2012; Cain, 2009; Heuvel-
Panhuizen, Middleton, & Streefland, 1995). Students need to understand how to find and use percentages in order to succeed in other topics and their careers.

Many students require an in depth explanation of what a percentage is. Chris Cain (2009) shares the lesson that finally helped her students understand percentages. This teacher was telling her students that when a percent is changed to a decimal fraction, the decimal point is just moved over two spaces. The teacher had taught this lesson multiple times before and the question of ‘why’ always seemed to surface. When a student asked the question the teacher decided to show the students why percents convert to decimals in this manner using ones, tens, and hundreds blocks. The first question she asked her students was why 45% and .45 were equal to each other. Her students could not come up with an answer so she asked her students what they were trying to take 45% of. Again, they could not respond. She continued to explain that ‘cent’ meant one hundred, and therefore, it was 45% of 100. She then had someone use the blocks to show her 45 out of 100 blocks. After she continued the lesson the students understood why these two quantities were equivalent. These students did not previously know what percentage meant and normally forgot that the rule they had been told was to move the decimal. After this lesson not one student forgot to move the decimal on the test.

Percentage misconceptions can effect students throughout their whole education and life. America’s Choice Mathematics Navigator (2012), an intervention program based on best practices, created a website which lists twelve common misconceptions that students have about percentages. Several of these reasons are similar to the decimal confusion discussed by Cain. Another common misconception that students hold is they do not realize that one whole is equal to 100%. This misconception can lead to confusion with the decimals. A few of the other misconceptions stem from questions asking to find the percent of some whole. Multiple students
do not know how to figure out what the ‘whole part’ is. A handful of students have trouble figuring out problems dealing with percent increases and decreases. They do not understand the concept of finding either the new total or simply what amount was added or subtracted from the total. Still, other misconceptions involve the general confusion about what operations can be used when dealing with percentages. Many students just try to subtract the percent decrease from the total. All of these misconceptions are developed because, in many cases, students do not have a solid base knowledge of percentages and do not fully understand the concept.

Another broad area that students face difficulties in is estimation.

**Estimation Misconceptions**

Estimation is used every day by students, teachers, adults, and children. The question of “how much” or “how long” makes an appearance in everyday life no matter what age or profession a person has. Estimation is also used in schools in every grade, but sometimes in the wrong way. Many students believe that estimating is a difficult task because they expect that they will be graded based on how close their estimation is to the correct answer (Muir, 2005). Research has shown that estimation should be used at every grade level and is an important technique to be able to demonstrate properly (Andre, Forrester, Gardner, Jones, Robertson, & Taylor, 2012; Colmer, 2006; Hunter & Towers, 2010; Muir, 2005). One of the biggest misconceptions students have about estimation is the idea that an estimation is just a guess (Muir, 2005).

In Hunter and Towers’ research (2010), students were given Cuisenaire rods and instructed to play with them. Cuisenaire rods are small rods which vary in length (1-10 cm) and color. They are used as manipulatives for students to demonstrate many concepts. The teacher, Jane, first gave the students multiple rods and had them simply play with the rods for about ten
minutes. She then asked the students if they could estimate how many orange rods it would take to cover one edge of the table from corner to corner. As Jane explained the question she used the words ‘estimate’ and ‘guess’ interchangeably. The constant interchanging might be the reason why one student said sixteen and another said nine and a half, which are not too close together in relation to the problem. Once the students’ estimates were all written on the board, Jane lined up the orange Cuisenaire rods along the table. She had the students count with her as she added one rod after another. When there were ten rods on the table they counted again and discussed if another rod would fit, which everyone agreed it would not. Jane answered by saying that the table can be estimated to be ten orange rods long. The students argued with her saying that the estimate should be ten and a half or ten and a quarter. She explained that when an estimate is being made the number can be rounded. It was apparent that the students did not fully understand what an estimate was; they thought it had to be exact.

Estimation can also be confusing and difficult for individuals who do not have a complete understanding of what units to use while estimating. Andre et al. (2012) proved that students were able to estimate in the English unit system much better than in the metric system. Their study required students to estimate the lengths of different items (dowel rods, puffy-paint line, a line, and a three-dimensional wooden cube) in both centimeters and inches. The results showed that students did exceedingly better when estimating in inches. Since this study was done in the United States, this was attributed to the fact that American students use inches as the primary unit of measure of lengths.

Students use various wrong strategies in order to estimate. David Hildreth (1983) provided the inappropriate strategies that students use when estimating measurements. One misconception students have when estimating area is to use some rectangular unit and estimate
each side using this arbitrary unit. Then, using the area formula, students calculate the area from the estimations. This approach obviously does not work because area always has a specific unit that is never rectangular. Another misconception that is held by students about measurement estimation is that they think they will be able to take a wild guess and come close to the answer. Many students do not understand what units each separate problem needs to be measured in, so they just pick one. Hildreth states that students are confused that the units the area should be measured with must be known before the estimation can be made. The concepts of percentages and estimation can be applied towards the concept of area; therefore, students must be able to comprehend all three subjects.

**Area Misconceptions**

**Teacher/preservice teacher misconceptions about area.** Many teachers and students struggle with the mathematical topic of area. Sherman and Randolph (2004), among others (e.g., Zacharos, 2006), have reported on the misunderstandings of students learning the concept of area. Possibly the biggest reason why students have trouble in this area is because their teachers do not fully understand the concept themselves. There are many teachers who do not know their content as well as they should; therefore, they are not able to teach it to their students in a comprehensible manner (Ball & Wilson, 1990; Bambico, 2002; Batroo, & Nason, 1996).

Other studies comment about the sole use of a formula when defining area. Yeo (2008) reported on a study of a fourth grade classroom where the teacher had the students define area as length times breadth. Although the students were able to computationally find the area of a square or rectangle, they were not able to actually define the term. In this study, the teacher accepted this formula as a definition and did not ask for a different one because even he was
unsure of the true definition of area. Yeo (2008) concluded that since the teacher in the study lacked the content knowledge, the students did not fully understand area either.

Other studies have been done that result in the same conclusions that Yeo (1998) made. Reinke (1997) analyzed seventy-six elementary preservice teachers’ abilities to look at the geometric figure in Figure 11 and describe a method to find the perimeter and the area of the shaded region.

**Figure 11.** Shaded Region Diagram Shown to Preservice Teachers from Reinke (1997).

When the preservice teachers completed the two questions, the types of strategies described were tallied. A portion of the subjects, 21.1%, completely ignored the semicircle and described the method to find the area of the whole rectangle. A smaller percentage, 15.8%, could not think of an answer and left the question blank while 10.4% of the preservice teachers offered other various incorrect strategies. Only about half of the subjects were able to describe a correct way to find the area of the shaded region. This result shows that these teachers are not completely prepared to teach this topic.

**Teacher/preservice teacher misconceptions between area and perimeter.** Teachers’ misconceptions do not end at area, they confuse area and perimeter as well. Many teachers teach these topics from a textbook at the same time which leads to student misunderstanding. Teachers do not clearly specify the difference and cannot fully explain the similarities, differences, and
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relationships between the two measurements (Kellogg, 2010; Leikin, Sinitsky, & Zazkis, 2013; Menon, 1998). The misconceptions that are held by teachers have serious implications for students’ understanding of these topics (Mewborn, 2001). A list of common teacher misconceptions is seen in Figure 12.

Studies have been done to provide information about teacher misconceptions. Amore and Pinilla (2006) investigated teachers’ misconceptions about the relationship between the perimeter and area of an object. The participants were given a three by three table with two inequalities in each entry. For example, the first box had two ‘greater than symbols’. In this instance, the teachers were supposed to find two objects where, when going from the first to the second, the perimeter increases and the area increases. Other entries in the table included less than symbols, for when the perimeter or area decreased, and equal signs, for when they stayed the same. Of the fourteen teachers who participated in this study, nine of them had a difficult time picturing and creating examples for all nine situations. The hardest examples for these teachers to generate were when the perimeter increased and the area decreased and vice versa. Nine teachers admitted that, at first, they thought these cases were impossible. As a result of this study, Amore and Pinilla (2006) inferred that many teachers believed that there was a correlation between the perimeter and area of an object.
A somewhat similar concept was studied by Ball and Wilson (1990). These researchers studied seven teachers of alternate education and twelve teacher education candidates. The subjects were given a scenario where a student came into the classroom excited because she found a theory that was not taught in class. She continued to say that, using a closed figure, as the perimeter increases, the area also increases. Following her claim, she showed a picture which she said proves her theory. The participants were then interviewed and asked if the student’s claim was right or not. The results showed that 57% of the alternate route teachers and 40% of the teacher education candidates recognized that this student’s claim was false. Ball and Wilson (1990) concluded that a large percentage of the teachers believed that there is a strong correlation between the perimeter of a figure and the area of a figure.

These findings are verified by Kathryn Reinke (1997). As discussed previously, the elementary preservice teachers were asked to explain how they would find the perimeter and area of the shaded region. In this study, 22.4% of the participants did the perimeter problem the same way they did the area problem. Reinke concluded that these preservice teachers could not state or show the difference between perimeter and area and thought that they were closely related.

Other research contributions to this topic were made by Henry and Soyibo (2006). They studied Jamaican preservice teachers’ performance and understanding of area and perimeter, as well as the relationship between them. The research included 200 student teachers who took an eighteen question multiple choice and four structured question test in order to judge their performance on the three topics. The preservice teachers were labeled as ‘barely satisfactory’, as the mean score was a 51.71%. The results also showed that the participants had the lowest understanding of the relationship between perimeter and area. Less than half of the subjects were able to figure out that if the sides of a square are doubled then the area is quadrupled. This
study proved that these Jamaican preservice teachers did not have a deep understanding of the relationship between area and perimeter.

**Student misconceptions about area.** Similarly to teachers, students are reported to hold misconceptions about calculating area (Sherman & Randolph, 2004; Zacharos, 2006). There are many students who can plug numbers into an area formula and obtain the correct answers but cannot give the actual meaning of the area of a figure. There has been an abundance of research done to prove this as well as to decipher where the misconceptions first originated (Byrd, 1983; Hirstein, Lamb, & Osborne, 1978; Matérn, 1989). Studies have also been done in order to discover how to fix these misunderstandings (Balomenou, 2006; Gratzer, 2000; Kordaki, 2003).

These findings were further researched by Sherman and Randolph (2004), who explored a fourth grade class who was given a pretest which asked what area means, how to measure it, and where it is found in life. After reviewing the students’ answers it was clear that they did not know what area was at all. Almost all of the students were able to recite the formula for area, but when asked to give a definition with words they were not able to do it. When asked what area was, there were answers as broad as ‘something in geometry’ and as wrong as ‘the space in a line’. When asked how to measure area, the students gave answers such as ‘using a ruler’, ‘using your hand’, and ‘using a formula’. The students were even more confused on where area is found in life. They said ‘in the book’, ‘in stories’, and ‘in a house’. Sherman and Randolph (2004) concluded that this test was a clear indication that these students had no real concept of what area actually was.

These conclusions were not only reached by Sherman and Randolph (2004). Zacharos (2006) conducted a study where fourth grade students were broken into two groups, a control group and an experimental group. The control group was taught area the conventional way, and
the experimental group attended an additional class where they were taught how to use different measuring tools. All of the students were then given the same problem. They were given a four by six centimeter rectangle as well as a square centimeter on their sheet of paper. The students were then asked to calculate the area of the rectangle. There were a few different strategies used and only 71.4% of the experimental group and 20% of the control group found the correct answer. Many other students tried to use a formula or claimed that the shape had no area. The participants were then asked how many of the unit squares fit into the rectangle. Approximately 60% of the control group and about 39% of the experimental group did not know the answer or wanted to look at the rectangle again. Zacharos (2006) inferred that the results to this question showed that students did not understand what the area of the figure actually means to the students.

Strategies that the students used when asked to find the area of other shapes, such as a trapezoid are also discussed by Zacharos (2006). Many participants tried to use the length times width formula for every single shape, even if it was not a regular shape. This result shows that they just memorized the formula but do not see how it should be applied. Another strategy that was used by many students was adding the base and height. This strategy shows that there is no understanding of area and what it is. The third most used strategy was finishing off figures. If the student was presented with an odd shape he/she would simply add to it in order to make it a shape he/she recognized and knew how to calculate the area of. Although the students in this study were able to use the trivial formula for basic shapes, Zacharos (2006) concluded that they did not understand any underlying concepts of area.

Similar results were obtained by Kidman (1999). He conducted a study where fourth, sixth, and eighth grade students were individually interviewed. They were shown rectangular
wooden blocks one at a time and asked to rate them on a scale of one through nineteen depending on how happy or upset they would be to get a chocolate bar that size. The ratings were then plotted and analyzed. It was found that nineteen out of thirty-six students saw area as the length plus the width, while sixteen students saw area as length times width. One student’s plot did not fit in with the other data. He believed that a rectangle with smaller dimensions had bigger area than a rectangle with bigger dimensions. More than half of these students did not have an accurate view on what area was.

**Student misconceptions between area and perimeter.** Since many textbooks teach perimeter and area together, most teachers do as well. The combining of these topics can lead to students confusing the two terms and their meanings. If the subjects are taught with enough emphasis on the difference between them, large numbers of students would use the two ideas interchangeably. Without a deep understanding of each concept, it is nearly impossible to truly understand the underlying similarities and differences. Research has shown that students have a difficult time seeing these differences and similarities, as well as the relationship between area and perimeter (Carle, 1993; Gough, 2004; Woodward, 1982). A list of common student misconceptions is shown in Figure 13.

**Figure 13. Student Misconceptions about Area.**

<table>
<thead>
<tr>
<th>Student Misconceptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Cannot give the definition of area or how to measure it</td>
</tr>
<tr>
<td>• Believe that the single ‘length times width’ formula works for every shape</td>
</tr>
<tr>
<td>• Some students believe the formula for area is ‘length plus width’</td>
</tr>
<tr>
<td>• Cannot describe the difference between area and perimeter</td>
</tr>
<tr>
<td>• Believe that perimeter and area are correlated</td>
</tr>
</tbody>
</table>
In the study by Amore and Pinilla (2006) discussed earlier, students of all ages were also asked to find examples for each of the nine entries in the table. It was found that when students were interviewed almost everyone believed there was a close relationship between the increase and decrease of the perimeter and the increase and decrease of the area of an object. More than half of the university students interviewed said that some of the nine cases were impossible. There were many students who had a very hard time switching their beliefs to the fact that there is no relationship. Of the students who did not have a problem changing their beliefs, many argued that the figures used in the examples shown were not geometric shapes and, therefore, they did not think they could use them. The results of this study showed that students of all ages believed that there is a strong correlation between perimeter and area.

Similarly to Amore and Pinilla, Naidoo and Naidoo (2008) studied seventh graders’ ability to calculate area and perimeter. The students were divided into two groups, the control group, which learned in a conventional classroom, and the experimental group, who learned using different technologies. The researchers concluded that the students who learned in a conventional classroom had a lower achieving rate than those who learned with technology. The students in the control group had high success rates when finding the area of a rectangle and square, but very low success rates, 45% and 5%, when finding the area of a parallelogram and triangle, respectively. When calculating the perimeter, the students in the control group had a high success rate when finding the perimeter of a square and triangle, but not for a rectangle and parallelogram. After the study was completed, the participants answered questionnaires. When asked does the area of a rectangle remain the same if the perimeter changes, 65% of the experimental group replied ‘yes’. When asked does the area of a rectangle change if the
perimeter changes, 80% of the control group said yes. This study shows that students have no understanding of the relationships between perimeter and area.

Similar results were reported by Kidman (1999). As discussed previously, about half of the students would have used a formula for perimeter in order to find the area. This result was very consistent throughout each grade, which shows that this misconception continued throughout each grade level. Through this study it is noted that students do not know the difference between the perimeter of an object and the area of an object.

The ability to closely estimate a shaded percentage of area is a skill that can only be developed with an understanding of the three topics. Many students struggle with making estimations because they do not want to be wrong. Area should be taught in more depth in schools so both teachers and students can eliminate the many misconceptions that are held. There is little research that links these three topics. Hence, the following study is aimed at finding where the misconceptions lie when making estimations based on percentage and area.

**Experimental Design and Data Collection**

This experiment was designed to test the hypothesis that students would have difficulties estimating the percentage of an area that is shaded and that students are more likely to overestimate than underestimate. During the study, students completed a ten-problem test that contained various geometric figures, each with a different percentage of shaded area. Three of the shapes also required an explanation for how the estimation was made. The test was evaluated to determine how close students’ estimates were and if they were generally overestimating or underestimating. Each problem was generated by involving all different percentages and altered methods of shading. This evaluation was compared to the results from a
brief survey that students filled out at the end of the assessment which asked which problems were easy or difficult, as well as how confident the student was in his/her estimates.

**Participants**

This study was conducted in both a high school and an university. A total of 74 students participated in this study. There were 46 high school student participants, 15 of these students were enrolled in a Geometry class and 31 were enrolled in a Pre-Calculus class. In the Geometry class 4 males and 11 females participated, one of whom had an IEP, and 1 student had a 504 plan. In the Pre-Calculus class 14 males and 17 females participated, from which 1 student had an IEP, and 3 students had a 504 plan. At the high school the student population is about 93% Caucasian, 2% African American, 2% Hispanic, and 3% Asian. Each class meets for 80 minutes every other day, with each math class consisting of an average of about 20 students. In order for these subjects to participate in this study, special permission was received. A consent form (see Appendices C and D) was given to each student in the classroom. If the student was willing to take part in the study, the student and his/her parents signed the consent form before the experiment took place. Additionally, the principal of the high school gave permission for the study to occur. Figure 14 lists the demographics of the high school participants.

**Figure 14. High School Student Demographics.**

<table>
<thead>
<tr>
<th>Gender</th>
<th># of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>18</td>
</tr>
<tr>
<td>Female</td>
<td>28</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Accommodations</th>
<th># of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEPs</td>
<td>2</td>
</tr>
<tr>
<td>504 Plans</td>
<td>4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Race of Whole School</th>
<th># of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Caucasian</td>
<td>93%</td>
</tr>
<tr>
<td>African American</td>
<td>2%</td>
</tr>
<tr>
<td>Hispanic</td>
<td>2%</td>
</tr>
<tr>
<td>Asian</td>
<td>3%</td>
</tr>
</tbody>
</table>
This instrument was also administered at a liberal arts university in the Northeast. There were 28 college student participants in this study, 11 males and 17 females. These students were enrolled in *Mathematics in Action* which is a mathematics course for non-majors that fulfills a College Core Curriculum (CCC) requirement for graduation from the university. This is the only mathematics course that many of these students take in their college career. Of the students who participated in the study, 25 were college freshmen and 3 were college sophomores. The university has a population of about 5,400 students consisting of approximately 5,100 undergraduate students and 300 graduate students. The majority of these students are from the surrounding counties in the state. The other portion of these students comes from different parts of the state, the United States, or foreign countries. The student population of this college is approximately 81% Caucasian, 4% African American, 5% Hispanic, 4% Asian, and 6% other.

The university has approximately 500 full-time and part-time instructional faculty. These college students also signed a consent form prior to completing the assessment (see Appendix E).

![Figure 15. College Student Demographics.](image)

<table>
<thead>
<tr>
<th>Gender</th>
<th># of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>11</td>
</tr>
<tr>
<td>Female</td>
<td>17</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Grade Level</th>
<th># of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freshman</td>
<td>25</td>
</tr>
<tr>
<td>Sophomore</td>
<td>3</td>
</tr>
<tr>
<td>Junior</td>
<td>0</td>
</tr>
<tr>
<td>Senior</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Race of Whole School</th>
<th># of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Caucasian</td>
<td>81%</td>
</tr>
<tr>
<td>African American</td>
<td>4%</td>
</tr>
<tr>
<td>Hispanic</td>
<td>5%</td>
</tr>
<tr>
<td>Asian</td>
<td>4%</td>
</tr>
<tr>
<td>Other</td>
<td>6%</td>
</tr>
</tbody>
</table>
15 lists the demographics of the college student participants.

Design

This experiment was designed to test the hypothesis that students struggle making estimations based on the percentage of area that is shaded in a shape. The first assessment was created based on research of area estimation and percentages. The researcher then piloted this original assessment in order to gain insights into what mistakes students were making and what questions should be asked in a different way. Once the assessment was piloted several times, it was altered for the intent of obtaining better data. The important question of how the estimate was made was inserted to get a better understanding of what students were thinking.

Before any students were allowed to participate in the study the appropriate consent forms were completed and returned. The high school students were required to get parent or guardian permission in order to participate. The parent consent forms were sent home with each student to be read and signed by a parent or guardian. The students were also required to complete a consent form, which was discussed in class prior to handing out the forms. The college students who participated also had to fill out a consent form. These forms were discussed in class before the students completed them. Figure 16 provides a list of the time line during the experiment.

Figure 16. Bulleted List of Time Line.

- Create the first assessment, pilot it to multiple individuals, and alter accordingly.
- Get consent forms signed by parents/guardians (high school) and all students.
- Give appropriate directions and have students complete the assessment.
The assessment given was a ten-problem test that showed ten different geometric figures which were all partially shaded in. On the test students were also asked to explain their thought process for three of the problems. Both the high school and college students took the ten-problem test in the last fifteen minutes of the class period. The assessment was followed by a five-question survey which asked students facts about themselves, such as gender and age, which question was easiest and hardest, and what area meant to them. The survey also asks each student to rate how accurate they thought their estimates were on a scale from one to ten, where one is ‘the least accurate’ and ten is ‘the most accurate’. The students were directed to complete the whole assessment within fifteen minutes. The proctor of the test explained to them that each estimate should be an educated estimate and should only take about half a minute to make. The proctor also explained that an estimate should be made for every problem. They were also directed to try to explain, to the best of their ability, how they made their estimate for the problems where this question was specified. The final direction was to complete the survey at the end completely and truthfully. Students were also reassured by the proctor that confidentiality would be kept through the entire study.

**Instrument Items and Justification**

The assessment was administrated to the students at the end of class. Students were given fifteen minutes to complete the test, which included the brief survey. The problems vary in difficulty and include all different percentages and methods of shading. Some problems had a portion less than 50% of the area shaded and other problems had percentages over 50% shaded. One problem had exactly 50% of the area shaded. There were problems that had one solid portion of the area shaded and problems that had multiple portions shaded. The students were
instructed to estimate what percentage of the whole area was shaded for each shape. Figure 17 provides the assessment problems and Figure 18 provides the follow-up survey questions.

**Figure 17. Instrument for High School and College-Level Study.**
**Figure 19.** Assessment Problem Properties.

<table>
<thead>
<tr>
<th>Problem Number</th>
<th>Under 50% Area Shaded</th>
<th>Exactly 50% Area Shaded</th>
<th>Over 50% Area Shaded</th>
<th>One Portion of Area Shaded</th>
<th>Multiple Portions of Area Shaded</th>
<th>Shade Line is Horizontal/Vertical</th>
<th>Shade Line is Not Horizontal/Vertical</th>
<th>Asks Student to Explain Process</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td></td>
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<tr>
<td>5</td>
<td>X</td>
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<td></td>
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<td>7</td>
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<td>X</td>
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<tr>
<td>8</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Each problem includes different aspects that affect the estimation. For example, Problem #1 has less than 50% of the area shaded, there is only one portion that is shaded, and one side of the portion that is shaded is slanted. On the other hand, Problem #9 has over 50% of the area shaded, there are multiple portions of the area that are shaded, the portions that are shaded are slanted, and the student is asked to explain his or her estimation.
The follow up-survey was meant to collect additional information from the participants. The first survey question collected demographic information to determine if age, gender, or major had an impact on performance. Questions #12 and #13 were meant to compare what problems the participants think are ‘easiest’ and ‘hardest’ with what problems are predicted to be ‘easiest’ and ‘hardest’. In Question #14 the participants were asked to define area in order to determine if they have a concrete understanding of area or not. The last question of the survey offered the participants the opportunity to express how well they think they did and reflect on their answers. Once the assessment was made and solidified, it was administered to the 74 participants.

**Methods of Data Analysis**

**Data Collection**

The test was graded based on how close the student’s estimation was to the correct solution. For each percentage point that the student was off, one point was given. The goal was to have the least amount of points. A score of 0 points would indicate that the student estimated each problem correctly, which was highly unlikely. Once each solution was graded, the researcher was also interested to know whether each solution was underestimated or overestimated. The results for each problem were tallied depending on whether the estimate was less than the correct percentage or greater than the correct percentage. It was also noted if each student was making their estimations based on quartiles. Every time a student answered with an exact quartile it was noted.

**Descriptive and Inferential Statistics**

Once the assessments were completely graded the results were compared based on distinguishing qualities to determine if there were significant differences in each group’s scores.
An analysis of variance (ANOVA) test on Minitab was used first to determine if gender had a significant difference on score. This test compared mean scores of males and females. An ANOVA test was also used to compare mean scores of each course (Geometry, Pre-Calculus, and Mathematics in Action) to test if a significant difference existed and give a p-value. Minitab was used to create a bar graph of over- and under-estimations to reveal if students did one or the other more frequently. The bar graph stated the exact numerical value of how many times students overestimated, underestimated, and estimated exactly correctly. A Minitab bar graph was also used to show how many students estimated in quartiles (25%, 50%, or 75%). An ANOVA test was used to see if there was a significant difference in scores depending on how much of the area was shaded (over or under 50%). This ANOVA test found means to compare and gave a p-value. The same type of ANOVA test was used to see if there was a significant difference based on if the figure had a horizontal/vertical shade line or not. The test found mean values to compare and provided a p-value. Lastly, an ANOVA test was done to test if there was a significant difference based on whether there was a single piece or multiple pieces of the geometric figure shaded.

The assessment also asked students to explain how they made their estimate for three problems. All of the students’ answers were broken into six different response categories and entered into Minitab. A bar chart was then used to determine which response the frequency of the responses. The same type of method was used to analyze the survey questions. Students were asked which problem they thought was ‘easiest’ and which was the ‘hardest’. This data was entered into Minitab and a bar chart was made to calculate numerical values of how many students responded with each problem number. The last survey question asked students how accurate they predicted their estimations to be. A mean value and mode was first found using
Minitab’s basic statistics feature and then a bar chart was created to find numerical values of each response. The use of these methods on Minitab allowed the hypothesis to be analyzed and key results to be deciphered.

Results

Quantitative Test Results

There were four main results that developed from this study:

1. *Students overestimated more frequently than underestimated and students did not frequently estimate in quartiles.*

   On the assessment, students overestimated many more times than they underestimated or got the correct answer (overestimate: 425 times, underestimate: 245 times, exact: 70 times). Also, students rarely made their estimations in quartiles (quartiles: 169 times, not quartiles: 571 times).

2. 
   a. *A significant difference in score did not exist based on the amount of area that was shaded (over or under 50%).*

      Students were not more likely to make a closer estimate if more than half of the shape was shaded, or vice versa (p-value: 0.533, under 50%-\(\bar{x}\): 5.797, 50% or over-\(\bar{x}\): 6.059).

   b. *There was a significant difference between scores based on whether the figure had a horizontal/vertical shade line or not.*

      Students were more likely to make a closer approximation if the shade line was horizontal/vertical rather than slanted (p-value: 0.00, horizontal/vertical-\(\bar{x}\): 1.975, slanted-\(\bar{x}\): 6.883).
c. There was a significant difference between scores based on if a single piece or multiple pieces of the shape were shaded.

Students were more likely to estimate accurately if a single piece of the shape was shaded instead of multiple pieces (p-value: 0.00, single piece-$\bar{x}$: 4.546, multiple pieces-$\bar{x}$: 7.935).

3. A significant difference existed between total scores and course.

Mathematics in Action and high school Pre-Calculus students were able to more accurately estimate the shaded percentage than high school Geometry students (p-value: 0.005, Mathematics in Action-$\bar{x}$: 59.88, Pre-Calculus-$\bar{x}$: 51.68, Geometry-$\bar{x}$: 72.50).

4. There was no significant difference between total scores for males and females.

Neither males nor females were closer to the correct percentage of area that was shaded on the assessment (p-value: 0.189, male-$\bar{x}$: 55.04, female- $\bar{x}$: 61.56).

Result 1: Students overestimated more frequently than underestimated and students did not frequently estimate in quartiles.

Students enrolled in all courses were more likely to overestimate than underestimate on the assessment. Out of a total of 740 responses, students overestimated 425 times, underestimated 245 times, and estimated exactly 70 times. Student’s solutions from each course were analyzed. It was found that in the Mathematics in Action course out of 280 total solutions, 156 were overestimated, 96 underestimated, and 28 solutions were exact. In the Pre-Calculus course out of 310 total solutions, 181 were overestimated, 102 underestimated and 27 solutions were exact. Lastly, in the Geometry course out of 150 total solutions, 88 were overestimated, 47 underestimated, and 15 solutions were exact. Figure 20 displays these results in a bar graph.
Each problem was also studied separately. Out of 10 problems, students overestimated most Frequently on Problem #8 (64% shaded, multiple pieces shaded, and the shade line was not horizontal/vertical), underestimated most often on Problem #7 (47% shaded, one piece shaded, and the shade line was not horizontal/vertical), and estimated exactly most frequently on problem 4 (50% shaded, one piece shaded, and the shade line was vertical). Figure 21 depicts these outcomes.
Analysis was also done on each problem to determine if students were likely to estimate in quartiles (i.e. 25%, 50%, or 75%). Results showed that students rarely estimated in quartiles. Out of a total of 740 answers, students only estimated quartiles 169 times, leaving 571 answers to not be quartiles. Out of the 169 quartile estimates, 58 of these estimates were actually correct (problem 4 had exactly 50% of the area shaded). Figure 22 presents a bar chart of these results by problem.

**Figure 22. Bar Chart of Quartile Estimates by Problem.**

**Result 2: a. A significant difference in score did not exist based on the amount of area that was shaded (over or under 50%).**

When performing an analysis of variance (ANOVA) to determine whether students were better able to estimate the percent of area that was shaded if the shaded area was over or under 50% of the total area, the p-value was 0.533. Therefore, evidence suggested that the percentage of area shaded was not a predictor of performance. The basic descriptive statistics gave a mean score of 5.797 points for the problems that had under 50% shaded and a mean score of 6.059 points for the problems that had 50% or over of the area shaded.
Result 2:  b. There was a significant difference between scores based on whether the figure had a horizontal/vertical shade line or not.

After performing an analysis of variance between the overall mean scores for problems that contained a horizontal/vertical shade line and problems which did not have a horizontal/vertical shade line, the p-value was 0.000. Hence, this evidence supports the claim that the orientation of the shade line is a predictor of performance. Further analysis revealed the mean total score of problems with a horizontal/vertical shade line was 1.975 points. The mean overall score of problems with a slanted shade line was 6.883 points. Since the goal of the assessment was to get a low score, this information suggests that students performed better on problems that had a horizontal/vertical shade line.

Result 2:  c. There was a significant difference between scores based on if a single piece or multiple pieces of the shape were shaded.

The analysis of variance done between the total mean scores for problems which had a single piece shaded and problems that had multiple pieces shaded revealed a p-value of 0.000. Thus, there was a significant difference between the performances on such problems and evidence suggests that the number of shaded pieces is a predictor of score. Supplementary analysis was implemented and this claim was further supported. The mean total score for problems that had only one piece of the shape shaded was 4.546 points and the mean total score for problems which had multiple pieces shaded was 7.935 points. These statistics suggest that students perform to a higher standard when only one piece of the shape is shaded.

Result 3: A significant difference existed between total scores and course.

Attention was also given to the results of the assessment based on the course each student was enrolled in. The test was given to 28 Mathematics in Action students (MATH 110: college
level), 31 Pre-Calculus students (11th and 12th graders), and 15 Geometry students (10th graders). The results illustrated that the *Mathematics in Action* and Pre-Calculus students were more precise with their estimations than the Geometry students. The mean total scores were examined and compared for each course: MATH 110-59.88 points, Pre-Calculus-51.68 points, and Geometry-72.50 points. Figure 23 compares the results of the assessment based on course, regardless of any other factors. The boxplot shows that the high school Geometry students had the highest scores and the largest range of scores. It also shows that the *Mathematics in Action* students had the lowest range of scores.

![Boxplot of Total Score by Course.](image)

An analysis of variance (ANOVA) between course and total mean score gave a p-value of 0.005.

**Result 4: There was no significant difference between total scores for males and females.**

Upon analysis of the total mean score based on gender, it was found that there was no statistically significant difference. Furthermore, neither males nor females were proven to estimate the percentage of area that is shaded in a given shape more accurately. The assessment was given to 29 males and 45 females. The mean total score for males was recorded as 55.04 points and the mean total score for females was recorded as 61.56 points. Figure 24 compares the results of the assessment by gender, regardless of what course they were enrolled in (male:
$n = 29$, female: $n = 45$). The boxplot shows that the mean score was higher for females than males. It also shows that the range of solutions given by females was larger than the range of solutions from males. This boxplot shows that males generally had less variability when giving estimations in the assessment.

**Figure 24. Boxplot of Total Score by Gender.**

An analysis of variance (ANOVA) between gender and total mean score revealed a p-value of 0.189. Therefore, this further proves the lack of significance based on gender.

**Analysis of Test Problems**

In order to determine why students had trouble with certain problems, each problem was examined in detail separately. Figure 25 is a table that lists each problem and if the problem had over or under 50% of the area shaded, if the shade line was straight or slanted, and if there was a single piece or multiple pieces of the area shaded. Additionally, the table ranks the question’s difficulty from the researcher’s point of view as well as by the results after the students finished the assessment. Problems categorized (by the researcher) as easy had a horizontal/vertical shade line and only had a single piece of area shaded. Problems categorized as moderate had a slanted shade line and only had a single piece of area shaded. Difficult level problems had a slanted shade line and were broken into multiple areas. There was a mix of under, over, and exactly
50% of the area shaded within each category. The researcher then determined what order to rank the questions in the same difficulty category by how familiar the shape was or how many pieces were shaded.

**Figure 25. Accuracy of Problems by Rankings**

<table>
<thead>
<tr>
<th>Problem</th>
<th>Under/Over/Exactly 50% Shaded</th>
<th>Horizontal/Vertical or Slanted Shade Line</th>
<th>Single/Multiple Pieces</th>
<th>Researcher’s Difficulty Rating</th>
<th>Student Results and Means(Ranked 1-10 Easiest-Hardest)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Diagram" /></td>
<td>Exact Vertical Single Easy</td>
<td>1</td>
<td>$\bar{x} = 0.696$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image2" alt="Diagram" /></td>
<td>Under Horizontal single Easy</td>
<td>2</td>
<td>$\bar{x} = 3.255$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image3" alt="Diagram" /></td>
<td>Under Slant Single Moderate</td>
<td>7</td>
<td>$\bar{x} = 6.829$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image4" alt="Diagram" /></td>
<td>Under Slant Single Moderate</td>
<td>3</td>
<td>$\bar{x} = 4.597$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image5" alt="Diagram" /></td>
<td>Under Slant Single Moderate</td>
<td>5</td>
<td>$\bar{x} = 5.446$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image6" alt="Diagram" /></td>
<td>over Slant Single Moderate</td>
<td>6</td>
<td>$\bar{x} = 6.459$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
As expected, the two problems that had a horizontal/vertical shade line proved to be the easiest for the students. In addition, except for one, the problems with shaded area broken into multiple pieces were the most difficult for the students. However, the researcher predicted the level of difficulty in the wrong order. Also, students did better on the problem that had multiple unshaded pieces than predicted, specifically; it was ranked as the fourth easiest problem based on the results from the assessment. Students had the most trouble with the estimation of Problem #8 (mean score-10.068) where the majority of students overestimated. This was surprising because Problem #10 (mean score-8.199) had two more pieces of the shape shaded.

The assessment had three problems where students were asked to explain how they obtained their solutions. The researcher sorted all of the students’ answers into six categories. The first category was ‘no response’. Out of 222 explanations, 51 of them were left blank. The most frequent response students gave was that they broke the shape up into pieces; they said that they split the shape in half, in quarters, or other sections (as shown in Figure 26).
This response was given 73 times. Another student response that was given was estimating the white space versus the black space which was given 10 times. The next response that was given often, 56 times out of 222, was that they just guessed. They did not explain any further how they made their estimate. Out of 222 answers, there were 27 times that students said they tried to estimate how many of the shaded shapes could fit into the whole figure (as shown in Figure 27).

**Figure 27. Student Explanation Sample.**

Lastly, five students stated that they tried to make the irregular figure into a more recognizable shape (as shown in Figure 28).
Figure 29 shows the breakdown of the method used by course. This pie chart shows that the high school Geometry students used the least number of methods. The high school Pre-Calculus students was the only class to use the method ‘made weird shape into normal shape’ and the *Mathematics in Action* students were the only ones to use the method ‘estimated the white space first’.

Figure 29. Pie Chart of Method By Course.
Analysis of Survey Questions

Once students completed the ten-problem test, they were asked to complete a five-question survey. The first question: state your age, gender, highest level of high school completed, and whether or not you are a math major, just asked students demographics. Thus, questions 2 through 5 will be looked at in more detail.

Question 2 asked students which problem they thought was the ‘easiest’ to estimate and why. Many students thought problem 4 (50% shaded, one piece shaded, vertical shade line) was the easiest problem to estimate. They commented that it appeared that exactly one half of the rectangle was shaded. One student said that the problem was easiest because “it is something I am familiar with and is the easiest to see that it is $\frac{1}{2}$ shaded”. Other students believed that Problem #3 (14% shaded, one piece shaded, horizontal shade line) was the easiest. A Pre-Calculus student said that it was the easiest because “it could be easily sectioned off”. Many students were using this type of method to estimate.

Question #3 asked students which problem they thought was the ‘hardest’ and why. Problems that had multiple pieces shaded were the ones that students said were the most difficult to estimate. The majority of students said that Problem #10 (39% shaded, multiple pieces shaded, slanted shade lines) was the hardest. One student wrote “it was such an irregular shape and the shaded area was broken up into pieces.” This response was one of the most common answers. Students were not able to estimate percentages easily because the shape was unfamiliar to them. Some students thought that Problem #9 (69% shaded, multiple pieces shaded, slanted shade lines) was the most difficult to estimate. A college student said “putting together a star to make a square in your head is very hard to do, which in turn makes it harder to guess the shaded
part.” This student had a method of estimating that she thought worked until she got to this problem.

Question 4 asked students what area means to them. The researcher believed that many students would not know how to describe area. Many students responded with the formula of area “length times width.” Other students attempted to use words in their explanation and described it as “the space on the inside of the shape.” There were also students who had less of an idea of how to define area and responded with answers such as “the volume inside of a shape,” “the entire shade area that is included within the shape,” and “area is the …” where the student didn’t even answer.

Finally, students were asked to estimate their accuracy on the entire ten-problem assessment on a scale from 1 to 10 (1 being the least accurate, 10 being the most). The mean accuracy estimation was 5.994. The most common answer was 6 (n=18) and the least common was 10 (n=0). The responses to a question such as this one is difficult to evaluate because it relies heavily on student opinion. One person might think that a score of 100 is very accurate while someone else might think that is very inaccurate. The overall mean score for all three courses combined was 59 points. Depending on what score is considered ‘accurate’ this mean score could match the predicted accuracy well or could be very incorrect. This survey question is extremely subjective to opinion.

The results that were found from the assessment problems and survey questions addressed the hypothesis statement. There were also results that emerged which were unexpected to the researcher. This study can provide great insight to mathematics teachers of all grades and can help students in the future.
Implications for Teaching

This experiment tested whether students would overestimate or underestimate when estimating area, as well as if they would just simply guess in quartiles. The survey taken at the end of the assessment also questioned subjects about the meaning of area. Based on student performance on the assessment and their comments on both the assessment and the survey, three suggestions for improvement in the classroom emerged.

Implication #1: Educators need to spend more time teaching and emphasizing estimation techniques.

While every student was able to make estimations for every problem on the assessment, their reasoning behind their estimations was all over the place. The majority of the explanations on the assessment were either “I guessed” or left completely blank. There were not many students who gave a legitimate estimation technique as an explanation answer. Based on the results of this study, it is clear that students are not comfortable making estimations. Teachers need to remind students that an estimation is just what it sounds like, an estimate and not a correct answer. Once students are content with this idea, they will be more likely to honestly attempt a more accurate estimation. Under the Common Core Standards, estimation is introduced in second grade to estimate lengths in standard units. Estimation continues to appear in each grade level. Hence, students learn estimation from a young age and should be more comfortable with it by the time they have reached high school.

When students have reached this point, it is important for teachers to enforce multiple correct estimation techniques that students can use inside and outside of the classroom. Educators should also have students estimate answers to various problems throughout each class period before actually solving the problem. Student’s confidence in their estimates will grow
over time as they become more comfortable with estimation techniques from doing them more often.

A great activity that could be done in class at any point of the day is to play a ‘Best Guess’ game. This can be done in a variety of topics. The teacher begins by stating or revealing a problem. The students should only have a few seconds to develop their estimation, or best guess, of the solution to the problem. The activity can ask students to estimate basic operations, multi-step operations, lengths, weights, areas, angles, and various other topics. Teachers can turn this activity into a game to see who can get closest to the solution, or as a bell ringer or ticket out the door. In order for the students to learn how to estimate more accurately, they should be asked to explain how they made their estimation frequently.

*Implication #2: Mathematics educators need to focus on teaching area in multiple ways and defining area clearly.*

Each participant in the study was asked what area means in the survey following the assessment. There were a plethora of answers ranging from the formula length times width, to correct or incorrect explanations using words, all the way to no answer at all. There are many students who, provided a rectangle with a given width and length, could find the correct area using a formula, but not all students could explain what that number represents. After reading the answers to the area question on the survey, this result was apparent.

Teachers of all grade levels need to teach area to their students in more than one way. Students should be able to explain what area means, not just be able to find it. Students will have a better chance of succeeding at this if they are taught area in multiple ways. Teachers need to focus on other methods rather than solely relying on textbook questions to teach area. Students should be provided with hands on activities. One of these activities could be measuring
shapes around the classroom to find perimeter and area. Students should also be given the opportunity to create their own area problems. When students are allowed to create their own problems it allows them to think on their own and it will show if they understand the concept or not. Once students have created a problem it is helpful for them to teach other students using their own problem. If students can explain problems correctly they more than likely have a good understanding of the material. Students should also work in groups to solve harder area problems so they can discuss and share ideas with their peers. Student understanding of area will increase by using multiple representations of area.

**Implication #3: College classes need to better prepare preservice teachers to teach area and estimation.**

One reason why students have a hard time estimating or explaining area is because their teachers are not completely familiar with the material themselves. College courses should be preparing preservice teachers to be experts in their subject area. Since estimation is not a technique that is learned that often in schools, college courses do not fully prepare pre-service teachers to teach it. If the teacher is not comfortable estimating, then the student has a very slim chance of making accurate estimates.

Area is another subject that teachers may not be fully prepared to teach to students. More than likely, these teachers were just taught a formula for area when they were in grade school. If a college course does not go more in depth of what area really is, it is likely that those teachers will just teach a formula to their students. This is a never-ending cycle that leads to a minimal understanding of the meaning of area. An incomplete understanding of area can then lead to an incomplete understanding of volume and surface area.
Suggestions for Future Research

Although the results of this study showed a significant difference in the course the subjects were enrolled in, a future study conducted with students in other grade levels could show even more conclusive results. It would be helpful if the study was conducted in every middle school and high school grade level, as well as multiple other college courses. It is possible that the students involved in this study in the higher level courses had more experience in estimating and they may have been more confident in their estimation techniques. A full study done throughout all grades could either solidify this possibility, or open new possibilities as to why one grade level makes more accurate estimations.

A future study should also focus on the number of males and females who are involved. Within the completed study, no grade level had the same number of males and females participating. The study showed that there was not a significant difference based on gender; however, a new study done with the same number of both genders could eliminate any bias the study has. A study done this way could not only produce new results based on gender, but could also change results based on the course subjects are taking.

In order to draw further conclusions about estimation techniques, the subjects involved in the study could be offered tools to help determine their estimation. Hands-on materials such as a ruler, an extra piece of paper, or extra assessment problems and scissors could produce different results. In a future study, subjects could be asked to complete some problems with no tools, some with a ruler, and some with any other tool the researcher would be interested in. The results could then be compared to see if the subjects made more accurate estimates or gave more in depth explanations using one tool over another, or no tool at all. This would give teachers
more specific information on how students think when making estimations and what techniques they should emphasize the most.

**Concluding Remarks**

Students do not use helpful estimation techniques and do not have a clear understanding of what area means. Teachers should make sure they have sufficient knowledge to teach these subjects and then emphasize them more in the classroom. Estimation skills are important not only in the classroom, but outside of it too. It is clear from the results of this study that students need more practice in both these subject matters. Although estimation and area may take longer to teach than planned, a complete understanding will aid students in higher grade levels and in life beyond school. It does not take long for a teacher to pause before solving any type of math problem and ask students to make a quick estimate of the answer. Short but frequent practice can improve a student’s ability to estimate more accurately.
References


Dahan, J. Revisiting the teaching of perimeter, area, and volume at a middle school level with Cabri environments. *IREM of Toulouse*.


Kellogg, M. Preservice elementary teachers' pedagogical content knowledge related to area and perimeter: A teacher development experiment investigating anchored instruction with web-based microworlds (2010). *Graduate School Theses and Dissertations*.


Appendix A

Assessment

For each of the following figures, estimate what percentage of the whole figure is shaded and answer the questions to the best of your ability.

1)  

Guess: ______% 

2)  

Guess: ______% 

3)  

Guess: ______% 

Explain how you made your estimate:
Estimations, Percentages, and Area

4) Guess: _______%

5) Guess: _______%

6) _______%

7) Guess: _______%

Explain how you made your estimate.
8) Guess: _______%

9) Guess: _______%

Explain how you made your estimate:

10) Guess: _______%
11) Fill in the following information about yourself:

Age: _________       Gender: __________       Math Major:  Yes or No

Highest level of school completed: ___________________________________________

12) Which question was the easiest to estimate? Why?

13) Which question was the hardest to estimate? Why?

14) What does area mean?

15) How accurate do you think your estimates are on a scale from 1 to 10 (1 being not accurate at all, 10 being extremely accurate)?
Appendix B

Assessment with Answers

1) [Diagram] 39%

2) [Diagram] 21%

3) [Diagram] 14%

4) [Diagram] 50%
5) 19%
6) 47%
8) 64%
9) 69%

10) 39%
Appendix C

TO: Parents/Guardians of Students in Geometry or Math 12
FROM: Kaitlyn Whitney
DATE: March 2014
RE: Consent Form

Purpose, Procedure, and Benefits

- The purpose of this study is to examine student ability to estimate area and determine what misconceptions about estimation, percentages, and area that students hold.

- Students will be given fifteen minutes to complete a 10 question assessment and a brief follow up survey. The assessment includes 10 shapes that are partially shaded in and the students will be asked to estimate what percent of the area of each shape is shaded. Three of the questions ask the students to explain how they made their estimation. The brief survey asks students questions about themselves (age, gender, race), what problem was easiest, what problem was hardest, what area means, and to rate their confidence level in their estimations on a scale from 1 to 10.

- The goal of this study is to determine what misconceptions students have regarding estimating, percentages, and area. This is an important study because it will help me and other teachers change our lessons in order to reduce these misconceptions.

Related Information

- Your student has been asked to participate in this study because he/she is a member of Mrs. Krakowiak/Miss Whitney’s Geometry or Pre-Calculus class.

- In order to maintain confidentiality, neither your student’s name nor yours will be used in any way during the study. Any name or identification will not be used with any materials related to the study.

- Participation in this study is voluntary; your student is free to withdraw from the study at any time. The choice to participate or not participate will not affect your student’s grade in any way.
There are no risks anticipated to the students. The safety of your student will be maintained per common school district practice, and is not likely to be impacted by this study.

The potential benefit to you and your student will be to receive more effective teaching strategies.

There is no cost to participate in this study.

Please read over and discuss the information with your student to make sure everyone is fully aware of everything involved in this study.

For any additional information or questions you may have, please feel free to contact me, Miss Whitney, by email: whitneyk@fredonia.edu.

You may also contact my college advisor, Dr. Keary Howard, at SUNY Fredonia by phone: 716-673-3873 or by email: keary.howard@fredonia.edu.

You may also contact the Human Subjects Protection Administrator, Maggie Bryan-Peterson, at SUNY Fredonia by phone: 716-673-3528 or by email: petersmb@fredonia.edu.

Thank you in advance for your participation in this important study. Please complete the attached consent form and return with your student. Remember, this form authorizes the use of data from your student’s assessment for the purposes of research.
Your participation in this important study is greatly appreciated. Please print and sign your name below to indicate your agreement for your child to participate in this study. You may retain a copy of this letter for your files. Thank you for giving this request your full consideration.

Voluntary Consent: I have read this memo. My signature below indicated that I freely agree to allow my son/daughter to participate in this study. If I withdraw my son/daughter from this study, I understand that there will be no penalty assessed to him/her. I understand that my son’s/daughter’s confidentiality will be maintained. I understand that if I have any questions about the study, I may contact Kaitlyn Whitney by email at whitneyk@fredonia.edu.

Please return this consent form as soon as possible. Thank you for your cooperation.

Parent/Guardian Name (please print): ____________________________________________

Parent/Guardian Signature: ____________________________________________________

Date: _______________

Parent/Guardian Email (optional): _______________________________________________
Appendix D

TO: Students in Geometry or Math 12
FROM: Miss Whitney
DATE: March 2014
RE: Consent Form

- You are being asked to participate in a research project.
- To participate you need to complete a 10 question assessment and a brief survey.
- By signing the consent form, you are allowing Miss Whitney to use your assessment information and data in the study.
- This assessment will be taken in the last 15 minutes of a normal class period.
- Your name will never be used in any way. The study will never identify you personally.
- Miss Whitney will keep your assessment. Your name will not be on the assessment anywhere.
- There are no risks involved in participating in this study. Your grade will not be affected by your decision to participate or not participate.
- You will not be given rewards for participation.
- If you feel uncomfortable at any time during the study you may withdraw without a penalty.
- Please remember that this study is an attempt to help improve lesson plans.

Please discuss this with your parent of guardian. If you or they have any questions, please feel free to contact me.

Please sign and return the original consent form as soon as possible.
STUDENT CONSENT FORM

SUNY Fredonia

Thank you for being a part of this study. Please print and sign your name in the space provided to show that you agree to participate. You may keep a copy of this form for your files.

**Voluntary Consent:** I have read this memo. My signature below shows that I freely agree to participate in this study. I understand that there will be no penalty for not participating. I understand that I may withdraw from this study at any time, also without a penalty. I understand that my name and any other personal information will be kept out of the study. I understand that if I have any questions about this study, I may contact Miss Whitney by email at whitneyk@fredonia.edu.

Please return this consent form as soon as possible. Thank you for your cooperation.

Student Name (please print): _______________________________________________________________

Student Signature: ___________________________________________________________________

Date: _______________
Appendix E

TO: Students in MATH 110
FROM: Kaitlyn Whitney
DATE: February 26, 2014
RE: Consent Form

Purpose, Procedure, and Benefits

- The purpose of this study is to examine student ability to estimate area and determine what misconceptions about estimation, percentages, and area that students hold.

- You will be given fifteen minutes to complete a 10 question assessment and a brief follow up survey. The assessment includes 10 shapes that are partially shaded in and the students will be asked to estimate what percent of the area of each shape is shaded. Three of the questions ask you to explain how you made your estimation. The brief survey asks you questions about yourself (age, gender, race), what problem was easiest, what problem was hardest, what area means, and to rate your confidence level in your estimations on a scale from 1 to 10.

- The goal of this study is to determine what misconceptions students have regarding estimating, percentages, and area. This is an important study because it will help me and other teachers change our lessons in order to reduce these misconceptions.

Related Information

- You have been asked to participate in this study to obtain data on college students.

- In order to maintain confidentiality, your name will be used in any way during the study. Any name or identification will not be used with any materials related to the study.

- Participation in this study is voluntary; you are free to withdraw from the study at any time. The choice to participate or not participate will not affect your grade in any way.

- There are no risks anticipated to you.

- The potential benefit to you will be to receive more effective teaching strategies.
There is no cost to participate in this study.

Please read over the information to become fully aware of everything involved in this study.

For any additional information or questions you may have, please feel free to contact me, Kaitlyn Whitney, by email: whitneyk@fredonia.edu.

You may also contact my college advisor, Dr. Keary Howard, at SUNY Fredonia by phone: 716-673-3873 or by email: keary.howard@fredonia.edu.

You may also contact the Human Subjects Protection Administrator, Maggie Bryan-Peterson, at SUNY Fredonia by phone: 716-673-3528 or by email: petersmb@fredonia.edu.

Thank you in advance for your participation in this important study. Please complete the attached consent form and return with your student. Remember, this form authorizes the use of data from your assessment for the purposes of research.
Thank you for being a part of this study. Please print and sign your name in the space provided to show that you agree to participate. You may keep a copy of this form for your files.

**Voluntary Consent:** I have read this memo. My signature below shows that I freely agree to participate in this study. I understand that there will be no penalty for not participating. I understand that I may withdraw from this study at any time, also without a penalty. I understand that my name and any other personal information will be kept out of the study. I understand that if I have any questions about this study, I may contact Miss Whitney by email at whitneyk@fredonia.edu.

Please return this consent form as soon as possible. Thank you for your cooperation.

Student Name (please print): ______________________________________________________

Student Signature: ______________________________________________________________

Date: _______________