Testing for a Poisson Mixture: Comparison of the Power of the Posterior Predictive Check (PPC) and Bootstrap Approaches

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Donghyung Lee

We, the dissertation committee for the above candidate for the

Doctor of Philosophy degree, hereby recommend

acceptance of this dissertation.

Nancy R. Mendell - Dissertation Advisor

Professor, Department of Applied Mathematics and Statistics

Stephen J. Finch - Chairperson of Defense

Professor, Department of Applied Mathematics and Statistics

Wei Zhu

Professor, Department of Applied Mathematics and Statistics

Yungtai Lo

Associate Professor, Department of Epidemiology and Population Health,

Albert Einstein College of Medicine, Yeshiva University

This dissertation is accepted by the Graduate School

Lawrence Martin

Dean of the Graduate School

Abstract of the Dissertation

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Identifying the number of components in a finite mixture is hard problem. Generally, the likelihood ratio test provides a robust method for statistical inference for this problem. However, the classical theorem for the asymptotic null distribution of the LRT statistic cannot be applied to finite mixture alternatives. So other inferential methods have been proposed to assess the statistical significance of an observed LRT value. Two such methods are the bootstrap and posterior predictive check (PPC). In this dissertation we conducted simulation studies to compare the power of the bootstrap method to the PPC method as it applies to identify the number of components in a Poisson mixture. We considered two simple hypothesis tests where we test a single Poisson distribution against a mixture of two Poisson distributions and a zero inflated Poisson (ZIP) distribution. For the two-component Poisson mixture alternative, we compared the power of the PPC method to the Bootstrap method. In the case of the zero inflated Poisson (ZIP) alternative, we compared the PPC method to the bootstrap method and two asymptotic tests

proposed by Rao and Chakravarti [20] and van den Broek [24] for detecting zero inflation in a Poisson. Simulated data sets were used to compare the performance of the methods for each test. A wide range of cases under these alternative hypotheses were considered with the objective of seeing whether one method is uniformly more powerful than the others for each of these alternatives.

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Chapter 1

Introduction

1.1 Determining the Number of Components in a Finite Mixture

Finite mixture models have been popular tools for modeling heterogeneous data; that is, data in which all observations do not come from the same distribution but instead are from a finite mixture of distributions. Since the number of components in the mixture is unknown, many researchers have been interested in methodologies for estimating the number of components in the mixture based on an observed sample. One popular approach is using a test of the null hypothesis that the observed data came from a mixture of k components against the alternative hypothesis that the data came from a mixture of $k+1$ components. Usually the hypothesis test is conducted based on the likelihood ratio test (LRT) statistic, $-2\log\psi(\underline{x})$, where $\psi(\underline{x}) = L(\hat{\underline{\theta}}_0 \mid \underline{x}) / L(\hat{\underline{\theta}}_1 \mid \underline{x})$ and $\hat{\underline{\theta}}_0$ and $\hat{\underline{\theta}}_1$ are the maximum likelihood estimates (MLE) vectors under the null and the alternative hypothesis respectively. Under regularity conditions, the LRT statistic asymptotically follows a chi-squared distribution with the degrees of freedom equal to the difference between the number of parameters in the null hypothesis and the number of parameters in the alternative hypothesis (Cox and Hinkley [3]). However, the regularity conditions of the classical theorem for the asymptotic null distribution of the LRT statistic do not hold in a test where the alternative is the finite mixture model since the mixing proportions lie on the boundary of the parameter space and the parameters are not identifiable under the null hypothesis.

To briefly explain the nature of the complications of the asymptotic null distribution of the LRT statistic, suppose that we wish to test a single normal distribution with mean $\mu = 0$ and unit variance (that is, $f(x; \mu = 0, \sigma^2 = 1)$) against a mixture of two normal distributions with means $\mu_1 = 0$ and μ_2 and common unit variances (that is, $\pi f(x; \mu_1 = 0, \sigma^2 = 1) + (1-\pi) f(x; \mu_2, \sigma^2 = 1)$) in a parameter space $\Omega = \{(\pi_1, \mu_2): [0,1] \times (-\infty, \infty)\}$ (McLachlan and Peel [16]). As one can see, the parameter vector (π_1, μ_2) under the null hypothesis lies on the boundary of the parameter space. Moreover, the mixing proportion π under the null hypothesis is not identifiable when $\mu_1 = \mu_2$. Comprehensive studies on the breakdown in the regularity conditions for the asymptotic theorem for the LRT statistic were done by Ghosh and Sen [9], and Hartigan [10].

Self and Liang [22] derived the asymptotic null distribution of the LRT statistic when the true parameter value is allowed to be on the boundary of parameter space. They showed that the asymptotic null distribution of the LRT statistic is a 50:50 mixture of chi-squared distributions with zero degree of freedom and one degree of freedom except in the case where a nuisance parameter is on the boundary of parameter space. However, there is a difficulty in directly applying this asymptotic property of the LRT statistic to testing the number of components in a finite mixture model since the asymptotic property varies over the parameters vector and its parameter space.

Meanwhile, many simulation studies were conducted to estimate the null distribution of the LRT statistic where the alternative is a mixture for assessing the number of components in a finite mixture. McLachlan [15] adapted the parametric bootstrap method to estimate a null distribution of the LRT statistic. He used this distribution to test a single normal distribution against a mixture of two normal distributions with equal within component variances. He showed that the bootstrap method can achieve a satisfactory empirical power for this test. On the basis of McLachlan's idea [15], Karlis and Xekalaki [11] introduced a sequential testing procedure to find out the number of components in a Possion mixture.

Thode et al. [23] simulated 7 percentile points (.10, .25, .50, .75, .90, .95, .99) of a distribution of the LRT statistic under the null hypothesis that an observed sample value arose from a single normal distribution versus the alternative that the sample value arose from a mixture of two normal distribution with a common variance. They compared them with percentile points of χ_1^2 and χ_2^2. Their results suggest that the empirical null distribution of the LRT statistic for the hypothesis test is closer to χ_2^2. By using these simulated percentile points as the null distribution of the LRT statistic for the same hypothesis test, Mendell et al. [17] estimated the sample sizes needed to obtain 50%, 80% and 90% power. In addition, they showed that power rate is reduced greatly when the mixing proportion, π, is outside of the interval from 0.2 to 0.8.

Lo et al. [14] showed that under suitable regularity conditions the asymptotic distribution of the log LRT statistic of the null hypothesis that the number of components in a mixture of k_0 normal distributions versus the alternative that the number of components in a mixture of k_1 normal distributions is a weighted sum of independent χ_1^2 random variables. This is referred to as "Lo-Mendell-Rubin Test". Lo [13] conducted a power comparison study on the Lo-Mendell-Rubin test, the bootstrap and posterior predictive check methods as applied to testing the number of components in a normal mixture with unequal variance within components. His simulation studies showed that the three tests have comparable powers.

1.2 Organization of the Dissertation

The main purpose of this dissertation is to compare the power of a Bayesian method for checking the suitability of the model to the observed data, called the posterior predictive check (PPC) method, to the bootstrap method as it applies to assess the LRT statistic. We consider two simple hypothesis tests:

Test A) The null hypothesis (H_0): the sample data arose from a single Poisson distribution ($k = 1$) ***versus*** the alternative hypothesis (H_{1A}): the sample data arose from a mixture of two Poisson distributions ($k = 2$).

Test B) The null hypothesis (H_0): the sample data arose from a single Poisson distribution ($k = 1$) ***versus*** the alternative hypothesis (H_{1B}): the sample data arose from a zero inflated Poisson (ZIP) distribution ($k = 2$).

The first test can be extended to test a mixture of k Poisson distributions versus a mixture of $k+1$ Poisson distributions as showed in Karlis and Xekalaki [11]. Since zero inflated Poission (ZIP) models are widely used in many scientific areas including epidemiology, psychiatry, and genetics, the second alternative hypothesis is important.

The probability density function of an observation x from a finite mixture of k Poisson distributions can be denoted by

$$p(x \mid \underline{\theta}) = \sum_{i=1}^{k} \pi_i \cdot \frac{e^{-\lambda_i} \cdot \lambda_i^x}{x!}, \ x = 0,1,2,3,\ldots$$

where k is the number of components, π_i is the mixing proportion of the i^{th} component with the constraint that $\sum_{i=1}^{k}\pi_i = 1$ and $\underline{\theta} = (\pi_1, \pi_2, ..., \pi_k, \lambda_1, \lambda_2, ..., \lambda_k)$. To avoid the lack of identifiability for the above mixture, we impose the restriction $0 < \lambda_1 < \lambda_2 < ... < \lambda_k$. Therefore, under H_0 the probability density function can be written as

$$p(x;\underline{\theta}_0) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}, \; x = 0,1,2,3,...$$

where $\underline{\theta}_0 = (\lambda)$ and under H_{1A} be written as

$$p(x \mid \underline{\theta}_{1A}) = \pi_1 \frac{e^{-\lambda_1} \cdot \lambda_1^{\,x}}{x!} + \pi_2 \frac{e^{-\lambda_2} \cdot \lambda_2^{\,x}}{x!}, \; x = 0,1,2,3,...$$

where $\underline{\theta}_{1A} = (\lambda_1, \lambda_2, \pi_1, \pi_2)$, $\pi_1 = 1 - \pi_2$, and $0 < \lambda_1 < \lambda_2$. The zero inflated Poisson distribution can be considered as a special case of the mixture of two Poisson distributions, where the mean of the first Poisson component is zero. Thus, under H_{1B} the model can be defined as follows:

$$p(x \mid \underline{\theta}_{1B}) = (1-\pi_2)\frac{e^{-\lambda_1} \cdot \lambda_1^{\,x}}{x!} + \pi_2 \frac{e^{-\lambda_2} \cdot \lambda_2^{\,x}}{x!}, \; x = 0,1,2,3,...$$

where $\underline{\theta}_{1B} = (\lambda_2, \pi_2)$ and $\lambda_1 = 0 < \lambda_2$.

For the two-component Poisson mixture alternative, we compared the PPC to the bootstrap method. In the case of the ZIP alternative, the PPC method was compared to the bootstrap method and two asymptotic tests proposed by Rao and Chakravarti [20] and van den Broek [24]. This power comparison study was conducted across different parameter settings and simulated data sets for each of these alternatives.

Chapter 2 presents the derivation of the likelihood ratio statistics and describes the EM algorithms for obtaining maximum likelihood estimates (MLE) of parameters under each of the alternatives. Chapter 3 introduces the four different statistical methods compared in our power analysis: the likelihood ratio tests (LRT) using (1) the parametric bootstrap and (2) the posterior predictive check methods as applied to simulate the null distribution of the LRT statistic and two asymptotic tests proposed by (3) Rao and Chakravarti [20] and (4) van den Borek [24] for detecting zero inflation in a Poisson distribution. In Chapter 4, we investigate the sensitivity of the power of the PPC method to the choice of parameters for the prior distribution and discuss results of the power comparison study. In Chapter 5, we draw conclusions and discuss possible future work.

Chapter 2

Likelihood Ratio Test for Poisson Mixture Models

2.1 Derivation of the Likelihood Ratio Test Statistic

Let $\underline{x} = (x_1, \cdots, x_n)$ denote the observed values of a random sample of size n from a Poisson mixture. Then the likelihood of the data is generally defined as

$$L(\underline{\theta} \mid x_1, ..., x_n) = L(\underline{\theta} \mid \underline{x}) = \prod_{i=1}^{n} p(x_i \mid \underline{\theta}).$$

Under the null hypothesis (H_0) that the observed data originated from a single Poisson distribution, the likelihood function, say L_0, is written as

$$L_0(\underline{\theta} \mid \underline{x}) = \prod_{i=1}^{n} p(x_i \mid \underline{\theta}) = \prod_{i=1}^{n} \left(\frac{e^{-\lambda} \cdot \lambda^{x_i}}{x_i!}\right). \tag{2.1}$$

Under the two-component Poisson mixture alternative (H_{1A}) that the observed data arose from a mixture of two Poisson distributions, the likelihood function, say L_{1A}, is written as

$$L_{1A}(\underline{\theta}_{1A} \mid \underline{x}) = \prod_{i=1}^{n} p(x_i \mid \underline{\theta}_{1A}) = \prod_{i=1}^{n} \left(\pi_1 \frac{e^{-\lambda_1} \cdot \lambda_1^{x_i}}{x_i!} + \pi_2 \frac{e^{-\lambda_2} \cdot \lambda_2^{x_i}}{x_i!}\right), \tag{2.2}$$

where $\underline{\theta}_{1A} = (\lambda_1, \lambda_2, \pi_1, \pi_2)$, $\pi_1 = 1 - \pi_2$, and $0 < \lambda_1 < \lambda_2$.

Under the zero inflated Poisson alternative (H_{1B}) that the observed data arose from a zero inflated Poisson distribution, the likelihood function, say L_{1B}, is written as

$$L_{1B}(\underline{\theta}_{1B} \mid \underline{x}) = \prod_{i=1}^{n} p(x_i \mid \underline{\theta}_{1B}) = \prod_{i=1}^{n} ((1-\pi_2)\frac{e^{-\lambda_1} \cdot \lambda_1^{x_i}}{x_i\,!} + \pi_2 \frac{e^{-\lambda_2} \cdot \lambda_2^{x_i}}{x_i\,!}), \qquad (2.3)$$

where $\underline{\theta}_{1B} = (\lambda_2, \pi_2)$ and $\lambda_1 = 0 < \lambda_2$.

The LRT statistic for testing $H_0: \underline{\theta} = \underline{\theta}_0$ against $H_1: \underline{\theta} = \underline{\theta}_1$ can be defined as

$$\psi(\underline{x}) = \frac{\sup_{H_0} L(\underline{\theta}_0 \mid \underline{x})}{\sup_{H_1} L(\underline{\theta}_1 \mid \underline{x})} = \frac{L(\hat{\underline{\theta}}_0 \mid \underline{x})}{L(\hat{\underline{\theta}}_1 \mid \underline{x})},$$

where $\hat{\underline{\theta}}_0$ is the MLE vector under H_0 and $\hat{\underline{\theta}}_1$ is the MLE vector under H_1. By using (2.1) and (2.2), we can derive the LRT statistic for test A as

$$\psi_A(\underline{x}) = \frac{\sup_{H_0} L_0(\underline{\theta}_0 \mid \underline{x})}{\sup_{H_{1A}} L_{1A}(\underline{\theta}_{1A} \mid \underline{x})} = \frac{L_0(\hat{\underline{\theta}}_0 \mid \underline{x})}{L_{1A}(\hat{\underline{\theta}}_{1A} \mid \underline{x})} = \frac{\prod_{i=1}^{n} (\frac{e^{-\hat{\lambda}} \cdot \hat{\lambda}^{x_i}}{x_i\,!})}{\prod_{i=1}^{n} (\hat{\pi}_1 \frac{e^{-\hat{\lambda}_1} \cdot \hat{\lambda}_1^{x_i}}{x_i\,!} + \hat{\pi}_2 \frac{e^{-\hat{\lambda}_2} \cdot \hat{\lambda}_2^{x_i}}{x_i\,!})},$$

where $\hat{\lambda}$ is the MLE of λ, where $\hat{\lambda} = \overline{x} = \sum_{i=1}^{n} x_i / n$, under H_0 and $\hat{\pi}_1, \hat{\pi}_2, \hat{\lambda}_1$, and $\hat{\lambda}_2$ are the MLEs of π_1, π_2, λ_1, and λ_2 under H_{1A}.

By using (2.1) and (2.3), the LRT statistic for test B can be derived as

$$\psi_B(\underline{x}) = \frac{\sup_{H_0} L_0(\underline{\theta}_0 \mid \underline{x})}{\sup_{H_{1B}} L_{1B}(\underline{\theta}_{1B} \mid \underline{x})} = \frac{L_0(\hat{\underline{\theta}}_0 \mid \underline{x})}{L_{1B}(\hat{\underline{\theta}}_{1B} \mid \underline{x})} = \frac{\prod_{i=1}^{n} (\frac{e^{-\hat{\lambda}} \cdot \hat{\lambda}^{x_i}}{x_i\,!})}{\prod_{i=1}^{n} ((1-\hat{\pi}_2) \frac{e^{-\lambda_1} \cdot \lambda_1^{x_i}}{x_i\,!} + \hat{\pi}_2 \frac{e^{-\hat{\lambda}_2} \cdot \hat{\lambda}_2^{x_i}}{x_i\,!})},$$

where $\hat{\lambda}$ is the MLE of λ, where $\hat{\lambda} = \overline{x} = \sum_{i=1}^{n} x_i / n$, under H_0, $\lambda_1 = 0$ and $\hat{\pi}_2$ and $\hat{\lambda}_2$ are the MLEs of π_2 and λ_2 under H_{1B}.

We propose to determine the MLEs of the parameters under the alternatives using the EM algorithm, and we will derive the EM algorithms for the mixture of two Poisson distributions and the zero inflated Poisson distribution in following section.

2.2 Application of the Expectation Maximization (EM) Algorithm

The Expectation-Maximization (EM) algorithm (Dempster et al. [4]) is broadly used to determine maximum likelihood estimates for incomplete data containing unobservable information under the alternative hypothesis that the component from which each observed value x_i has arisen is not known.

To define the incompleteness of the data mathematically, we need an additional variable z_i. The variable z_i is a vector of zero-one indicator variables associated with the data value x_i that specifies from which component of the mixture the data value x_i is generated. Under each of our alternatives, it is denoted by a two-dimensional vector $z_i = (z_{i1}, z_{i2})$. For instance, in the case that the observed data value x_i arose from the first component of the mixture, the first vector element z_{i1} becomes 1 and the second vector element z_{i2} becomes 0. The vector of these zero-one indicator vectors can be written as $\underline{z} = (z_1, \cdots, z_n)$.

In fact, we want to find the MLEs that maximize the incomplete log likelihood $\log L_1(\underline{\theta}_1; \underline{x})$. It is, however, well-known that it is difficult to calculate the MLEs of the incomplete log likelihood $\log L_1(\underline{\theta}_1; \underline{x})$ directly. In addition, it has already been shown that the MLEs of the expected complete log likelihood $E[\log L_1(\underline{\theta}_1, \underline{z}; \underline{x})]$ maximize the incomplete log likelihood $\log L_1(\underline{\theta}_1; \underline{x})$ (Neal and Hinton [19]). Therefore, we use the expected complete log likelihood $E[\log L_1(\underline{\theta}_1, \underline{z}; \underline{x})]$ for the maximization procedure instead of the incomplete log likelihood

$\log L_1(\underline{\theta}_1;\underline{x})$.

To determine the MLEs that maximize $E[\log L_1(\underline{\theta}_1,\underline{z};\underline{x})]$, we use the EM algorithm framework that consists of two steps, the Expectation(E)-step and the Maximization(M)-step. The E-step calculates the expectations of z_{ij} with respect to the posterior density of $\underline{z}$. Note that in the first iteration of the EM algorithm the E-step uses an initial parameter value $\underline{\theta}_1^{(0)}$ usually chosen based on the observed data $\underline{x}$. The M-step updates $\underline{\theta}_1^{(k+1)}$ that maximizes the expectation of the complete log-likelihood. These two steps are repeated until the difference in successive log likelihood is less than a specified small value τ, $\log L_1(\underline{\theta}_1^{(k+1)};\underline{x}) - \log L_1(\underline{\theta}_1^{(k)};\underline{x}) < \tau$. We use $\tau = 10^{-5}$.

2.2.1 EM Algorithm for obtaining the MLEs of Parameters of the Two-component Poisson Mixture Distribution

The complete log-likelihood $\log L_{1A}(\underline{\theta}_{1A},\underline{z};\underline{x})$ under H_{1A} can be derived as follows:

$$\log L_{1A}(\underline{\theta}_{1A},\underline{z};\underline{x}) = \log p(\underline{x},\underline{z};\underline{\theta}_{1A})$$

$$= \log \prod_{i=1}^{n} [\, p(x_i, z_i;\underline{\theta}_{1A})]$$

$$= \log \prod_{i=1}^{n} \prod_{j=1}^{2} [\, p(x_i \mid z_{ij} = 1;\underline{\theta}_{1A}) \cdot p(z_{ij} = 1)]^{z_{ij}}$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{2} \left(z_{ij} \cdot \log p(x_i \mid z_{ij} = 1;\underline{\theta}_{1A}) + z_{ij} \cdot \log \pi_j \right).$$

The expectations of z_{ij} with respect to the posterior density of $\underline{z}$, $p(\underline{z} \mid \underline{x};\underline{\theta}_{1A})$, can be derived as follows:

$$E(z_{ij} \mid x_i; \underline{\theta}_{1A}^{(k)})_{p(\underline{z}|\underline{x};\underline{\theta}_{1A})}$$

$$= \frac{p(x_i \mid z_{ij} = 1; \underline{\theta}_{1A}^{(k)}) \cdot \pi_j^{(k)}}{\sum_{m=1}^{2} p(x_i \mid z_{im} = 1; \underline{\theta}_{1A}^{(k)}) \cdot \pi_m^{(k)}}$$

$$= \frac{\dfrac{e^{-\lambda_j^{(k)}} \cdot (\lambda_j^{(k)})^{x_i}}{x_i!} \cdot \pi_j^{(k)}}{\dfrac{e^{-\lambda_1^{(k)}} \cdot (\lambda_1^{(k)})^{x_i}}{x_i!} \cdot \pi_1^{(k)} + \dfrac{e^{-\lambda_2^{(k)}} \cdot (\lambda_2^{(k)})^{x_i}}{x_i!} \cdot \pi_2^{(k)}},$$

where $\underline{\theta}_{1A}^{(k)} = (\lambda_1^{(k)}, \lambda_2^{(k)}, \pi_1^{(k)}, \pi_2^{(k)})$.

The expectation of the complete log-likelihood $\log L_{1A}(\underline{\theta}_{1A}, \underline{z}; \underline{x})$ is as follows:

$$E[\log L_{1A}(\underline{\theta}_{1A}, \underline{z}; \underline{x})]$$
$$= \sum_{i=1}^{n} \sum_{j=1}^{2} \left(E(z_{ij} \mid x_i; \underline{\theta}_{1A}) \cdot \log p(x_i \mid z_{ij} = 1; \underline{\theta}_{1A}) + E(z_{ij} \mid x_i; \underline{\theta}_{1A}) \cdot \log \pi_j \right) \tag{2.4}$$

Differentiating equation (2.4) with respect to λ_j, we get

$$\frac{\partial E[\log L_{1A}(\underline{\theta}_{1A}, \underline{z}; \underline{x})]}{\partial \lambda_j}$$
$$= -\sum_{i=1}^{n} E(z_{ij} \mid x_i; \underline{\theta}_{1A}) + \frac{\sum_{i=1}^{n} E(z_{ij} \mid x_i; \underline{\theta}_{1A}) \cdot x_i}{\lambda_j}. \tag{2.5}$$

Solving the normal equation generated from equation (2.5) for λ_j, we get the equation for updating λ_j as

$$\lambda_j^{(k+1)} = \frac{\sum_{i=1}^{n} E(z_{ij} \mid x_i; \underline{\theta}_{1A}^{(k)}) \cdot x_i}{\sum_{i=1}^{n} E(z_{ij} \mid x_i; \underline{\theta}_{1A}^{(k)})}, \quad j = 1, 2$$

To get the update equation of π_j, we must impose the restriction $\sum_{j=1}^{2}\pi_j = 1$ on the equation (2.4). For this reason we use the Lagrange multiplier α and formulate the equation as follows:

$$G(\underline{\theta}_{1A}) = E[\log L_{1A}(\underline{\theta}_{1A}, \underline{z}; \underline{x})] - \alpha(\sum_{j=1}^{2}\pi_j - 1) \tag{2.6}$$

Differentiating equation (2.6) with respect to π_j, we get

$$\begin{aligned}\frac{\partial G(\underline{\theta}_{1A})}{\partial \pi_j} &= \frac{\partial}{\partial \pi_j}\left(E[\log L_{1A}(\underline{\theta}_{1A}, \underline{z}; \underline{x})] - \alpha(\sum_{j=1}^{2}\pi_j - 1)\right) \\ &= \frac{1}{\pi_j}\sum_{i=1}^{n} E(z_{ij} \mid x_i; \underline{\theta}_{1A}) - \alpha.\end{aligned} \tag{2.7}$$

Multiplying equation (2.7) by π_j and setting the partial to zero, we get

$$\sum_{i=1}^{n} E(z_{ij} \mid x_i; \underline{\theta}_{1A}) - \alpha \cdot \pi_j = 0. \tag{2.8}$$

By taking summation on equation (2.8) over all components, we get

$$\sum_{j=1}^{2}\sum_{i=1}^{n} E(z_{ij} \mid x_i; \underline{\theta}_{1A}) - \alpha \cdot \sum_{j=1}^{2}\pi_j = \sum_{j=1}^{2}\sum_{i=1}^{n} E(z_{ij} \mid x_i; \underline{\theta}_{1A}) - \alpha = 0 \quad \left(\because \sum_{j=1}^{2}\pi_j = 1\right).$$

Solving this quantity for the Lagrange multiplier α, we get

$$\alpha = \sum_{j=1}^{2}\sum_{i=1}^{n} E(z_{ij} \mid x_i; \underline{\theta}_{1A}) = n. \tag{2.9}$$

Substituting this quantity (2.9) into equation (2.7) and solving (2.7) for π_j, we get the following update equation for π_j

$$\pi_j^{(k+1)} = \frac{\sum_{i=1}^{n} E(z_{ij} \mid x_i; \underline{\theta}_{1A}^{(k)})}{n}, \quad j = 1, 2$$

The summary of the EM algorithm for the mixture of two Poisson distributions is shown in Table 2.1.

Table 2.1: The EM algorithm for obtaining the MLEs of $\underline{\theta}_{1A} = (\lambda_1, \lambda_2, \pi_1, \pi_2)$

1. Choose initial estimates for $\underline{\theta}_{1A}^{(0)} = (\lambda_1^{(0)}, \lambda_2^{(0)}, \pi_1^{(0)}, \pi_2^{(0)})$ based on the observed data $\underline{x}$.

2. [E-step] For i=1,2,…,n; j=1,2

$$E(z_{ij} \mid x_i; \underline{\theta}_{1A}^{(k)})_{p(\underline{z}|\underline{x};\underline{\theta}_{1A})} = \frac{\dfrac{e^{-\lambda_j^{(k)}} \cdot (\lambda_j^{(k)})^{x_i}}{x_i!} \cdot \pi_j^{(k)}}{\dfrac{e^{-\lambda_1^{(k)}} \cdot (\lambda_1^{(k)})^{x_i}}{x_i!} \cdot \pi_1^{(k)} + \dfrac{e^{-\lambda_2^{(k)}} \cdot (\lambda_2^{(k)})^{x_i}}{x_i!} \cdot \pi_2^{(k)}}$$

3. [M-step] For j=1,2

$$\lambda_j^{(k+1)} = \frac{\sum_{i=1}^{n} E(z_{ij} \mid x_i; \underline{\theta}_{1A}^{(k)}) \cdot x_i}{\sum_{i=1}^{n} E(z_{ij} \mid x_i; \underline{\theta}_{1A}^{(k)})}$$

$$\pi_j^{(k+1)} = \frac{\sum_{i=1}^{n} E(z_{ij} \mid x_i; \underline{\theta}_{1A}^{(k)})}{n}$$

4. Repeat steps 2 and 3 until the value the parameter $\underline{\theta}_{1A}^{(k+1)} = (\lambda_1^{(k+1)}, \lambda_2^{(k+1)}, \pi_1^{(k+1)}, \pi_2^{(k+1)})$ converges.

2.2.2 EM Algorithm for obtaining the MLEs of Parameters of the Zero Inflated Poisson Distribution

The zero inflated Poisson model can be viewed as the two-component Poisson mixture model, where the mean of the first component is zero. Thus, all procedures to derive EM algorithm obtaining the MLEs of parameters of the zero inflated Poisson are the same as that we presented in 2.3.1. In the EM algorithm for a zero inflated Poisson distribution, the E step does not need to calculate the expectations of z_{i1} because we already know that λ_1 is equal to zero. The E-step calculates only the expectations of z_{i2} and the equation can be derived as follows:

$$E(z_{i2} \mid x_i; \underline{\theta}_{1B}^{(k)})_{p(\underline{z}|\underline{x};\underline{\theta}_{1B})}$$

$$= \frac{p(x_i \mid z_{i2}=1; \underline{\theta}_{1B}^{(k)}) \cdot \pi_2^{(k)}}{\sum_{m=1}^{2} p(x_i \mid z_{im}=1; \underline{\theta}_{1B}^{(k)}) \cdot \pi_m^{(k)}}$$

$$= \frac{\dfrac{e^{-\lambda_2^{(k)}} \cdot (\lambda_2^{(k)})^{x_i}}{x_i!} \cdot \pi_2^{(k)}}{\dfrac{e^{-\lambda_1} \cdot (\lambda_1)^{x_i}}{x_i!} \cdot (1-\pi_2^{(k)}) + \dfrac{e^{-\lambda_2^{(k)}} \cdot (\lambda_2^{(k)})^{x_i}}{x_i!} \cdot \pi_2^{(k)}}$$

$$= \begin{cases} \dfrac{\pi_2^{(k)} \cdot e^{-\lambda_2^{(k)}}}{(1-\pi_2^{(k)}) + \pi_2^{(k)} \cdot e^{-\lambda_2^{(k)}}} & , when\ x_i = 0 \\ 1 & , when\ x_i > 0 \end{cases},$$

where $\underline{\theta}_{1B}^{(k)} = (\lambda_2^{(k)}, \pi_2^{(k)})$ and $\lambda_1 = 0$.

The M-step updates λ_2 and π_2 by using the expectations of z_{i2} calculated from the E step. The equations for updating λ_2 and π_2 are as follows:

$$\lambda_2^{(k+1)} = \frac{\sum_{i=1}^{n} E(z_{i2} \mid x_i; \underline{\theta}_{1B}^{(k)}) \cdot x_i}{\sum_{i=1}^{n} E(z_{i2} \mid x_i; \underline{\theta}_{1B}^{(k)})}, \quad \pi_2^{(k+1)} = \frac{\sum_{i=1}^{n} E(z_{i2} \mid x_i; \underline{\theta}_{1B}^{(k)})}{n}$$

The summary of the EM algorithm for the zero inflated Poisson distribution is shown in Table 2.2.

Table 2.2: The EM algorithm for obtaining the MLEs of $\underline{\theta}_{1B} = (\lambda_2, \pi_2)$

1. Choose initial estimates for $\underline{\theta}_{1B}^{(0)} = (\lambda_2^{(0)}, \pi_2^{(0)})$ based on the observed data $\underline{x}$.

2. [E-step] FOR i=1,…,n

$$E(z_{i2} \mid x_i; \underline{\theta}_{1B}^{(k)})_{p(\underline{z}|\underline{x};\underline{\theta}_{1B})} = \begin{cases} \dfrac{\pi_2^{(k)} \cdot e^{-\lambda_2^{(k)}}}{(1-\pi_2^{(k)}) + \pi_2^{(k)} \cdot e^{-\lambda_2^{(k)}}}, & when\ x_i = 0 \\ 1, & when\ x_i > 0 \end{cases}$$

3. [M-step]

$$\lambda_2^{(k+1)} = \frac{\sum_{i=1}^{n} E(z_{i2} \mid x_i; \underline{\theta}_{1B}^{(k)}) \cdot x_i}{\sum_{i=1}^{n} E(z_{i2} \mid x_i; \underline{\theta}_{1B}^{(k)})}$$

$$\pi_2^{(k+1)} = \frac{\sum_{i=1}^{n} E(z_{i2} \mid x_i; \underline{\theta}_{1B}^{(k)})}{n}$$

4. Repeat steps 2 and 3 until the value of the parameter $\underline{\theta}_{1B}^{(k+1)} = (\lambda_2^{(k+1)}, \pi_2^{(k+1)})$ converges.

Chapter 3

Statistical Methods

3.1 Bootstrap Methods

3.1.1 Parametric and Nonparametric Bootstrap Methods

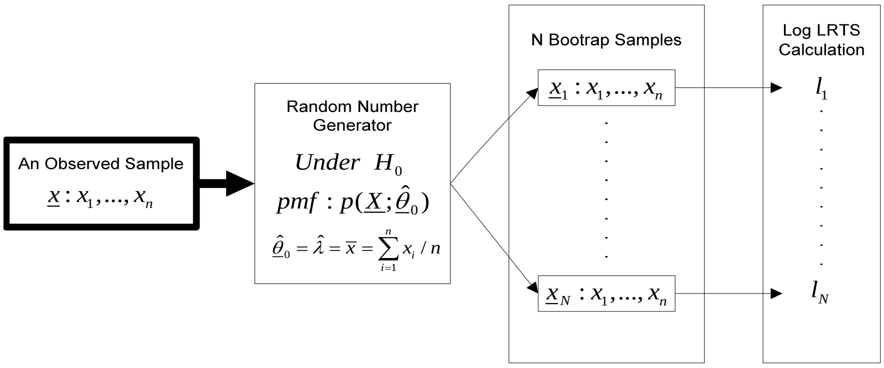
The bootstrap enables us to estimate the distribution of the statistic based on the observed sample values. The bootstrap procedure involves collecting many replications of the observed sample. For each replicated sample a replication of the statistic is generated. On the basis of these replications of the statistic an empirical distribution of the statistic is constructed. Using this empirical distribution we can conduct a hypothesis test. That is, the bootstrap method provides statistical inference for a single observed sample.

The bootstrap method was introduced by Efron [5]. It has two different types of approaches referred to as the "nonparametric" and "parametric" approaches. The "nonparametric" bootstrap approach does not require any distribution assumptions. Thus, the "nonparametric" bootstrap is distribution-free. It involves collecting N replications of size n by sampling with replacement from the sample observations. These replications are referred to as "nonparametric" bootstrap samples.

In the "parametric" bootstrap approach, we assume the null distribution. On the basis of this assumption we calculate the MLEs of parameters of the distribution from which we take our bootstrap samples. Bootstrap samples that have the same size n as the observed sample are generated from this null distribution.

In our context, it is assumed that the observed sample comes from a single Poisson distribution, under the null hypothesis. The MLE $\hat{\lambda}$ of the parameter λ of the Poisson distribution is calculated based the observed sample. The Poisson distribution using the MLE of λ (i.e., the sample average in this case) as its parameter is used to generate N bootstrap samples. For each generated bootstrap sample, we calculate the LRT statistic. These replicated LRT statistic values are used to estimate a null distribution of the LRT statistic. For our simulation study, we use the "parametric" bootstrap approach. The re-sampling procedure using the parametric bootstrap method can be summarized with Figure 3.1.

Figure 3.1: The flow chart of the parametric bootstrap method



3.1.2 Empirical Type I Error Rate of the Parametric Bootstrap

The type I error rate is the probability of rejecting the null hypothesis when it is true. To obtain an empirical type I error rate for the parametric bootstrap method, we generate M samples of size n under the null hypothesis. For each sample we conduct the hypothesis test using the parametric bootstrap and count the number of times we reject the null hypothesis. The number of rejections divided by the number of samples M is the empirical type I error rate of the test.

The steps to calculate the empirical type I error rate of the hypothesis test A and B using the parametric bootstrap method are as follows: (For the hypothesis test A, we follow Step 2A. For the hypothesis test B, we follow Step 2B.)

Step 1. Generate M samples of size n from a single Poisson distribution, under the null hypothesis.

Step 2A. For each observed sample, calculate the log LRT statistic, say l_{obs}, as

$$l_A = -2\log\psi_A(\underline{x}) = -2\log\frac{\sup\limits_{H_0} L_0(\underline{\theta}_0;\underline{x})}{\sup\limits_{H_{1A}} L_{1A}(\underline{\theta}_{1A};\underline{x})} = -2\log\frac{\prod_{i=1}^{n}\left(\frac{e^{-\hat{\lambda}}\cdot\hat{\lambda}^{x_i}}{x_i!}\right)}{\prod_{i=1}^{n}\left(\hat{\pi}_1\frac{e^{-\hat{\lambda}_1}\cdot\hat{\lambda}_1^{x_i}}{x_i!}+\hat{\pi}_2\frac{e^{-\hat{\lambda}_2}\cdot\hat{\lambda}_2^{x_i}}{x_i!}\right)} \qquad (3.1)$$

where $\hat{\lambda}$ is the MLE of λ, where $\hat{\lambda} = \bar{x} = \sum_{i=1}^{n} x_i / n$, under H_0 and $\hat{\pi}_1, \hat{\pi}_2, \hat{\lambda}_1$, and $\hat{\lambda}_2$ are the MLEs of π_1, π_2, λ_1, and λ_2 under H_{1A}.

Step 2B. For each observed sample, calculate the log LRT statistic, say l_{obs}, as

$$l_B = -2\log\psi_B(\underline{x}) = -2\log\frac{\sup\limits_{H_0} L_0(\underline{\theta}_0;\underline{x})}{\sup\limits_{H_{1B}} L_{1B}(\underline{\theta}_{1B};\underline{x})} = -2\log\frac{\prod_{i=1}^{n}\left(\frac{e^{-\hat{\lambda}}\cdot\hat{\lambda}^{x_i}}{x_i!}\right)}{\prod_{i=1}^{n}\left((1-\hat{\pi}_2)\frac{e^{-\lambda_1}\cdot\lambda_1^{x_i}}{x_i!}+\hat{\pi}_2\frac{e^{-\hat{\lambda}_2}\cdot\hat{\lambda}_2^{x_i}}{x_i!}\right)}, \qquad (3.2)$$

where $\hat{\lambda}$ is the MLE of λ, where $\hat{\lambda} = \bar{x} = \sum_{i=1}^{n} x_i / n$, under H_0, $\lambda_1 = 0$ and $\hat{\pi}_2$ and $\hat{\lambda}_2$ are the MLEs of π_2 and λ_2 under H_{1B}.

Step 3. Collect N parametric bootstrapped samples on the basis of the observed sample and for each bootstrapped sample calculate the log LRT statistic, say l_i, $i = 1,...,N$.

Step 4. Construct the empirical null distribution using these values of the log LRT statistics and find the $(1-\alpha)$ percentile of it. The percentile value is denoted by l_α.

Step 5. Compare l_{obs} and l_α. If l_{obs} is bigger than l_α then we reject the null hypothesis.

Step 6. Count the number of times we reject the null hypothesis over M samples and divide it by the number of samples M. The calculated value is the empirical type I error rate of the test.

3.1.3 Empirical Power of the Parametric Bootstrap

Power is defined as the probability of rejecting the null hypothesis when the alternative is true. Thus, to get an empirical power for the method M samples of size n are generated under each of the alternative hypotheses.

The steps to simulate the power of the hypothesis test A and B using the parametric bootstrap method are as follows: (For the hypothesis test A, we follow Step 1A and 2A. For the hypothesis test B, we follow Step 1B and 2B.)

Step 1A. Generate M samples of size n from a mixture of two Poisson distributions, under H_{1A}.

Step 1B. Generate M samples of size n from a zero inflated Poisson distribution, under H_{1B}.

Step 2A. For each observed sample, calculate the log LRT statistic, say l_{obs}, as (3.1)

Step 2B. For each observed sample, calculate the log LRT statistic, say l_{obs}, as (3.2)

Step 3. Collect N parametric bootstrapped samples on the basis of the observed sample and for each bootstrapped sample calculate the log LRT statistic, say l_i, $i = 1, ..., N$.

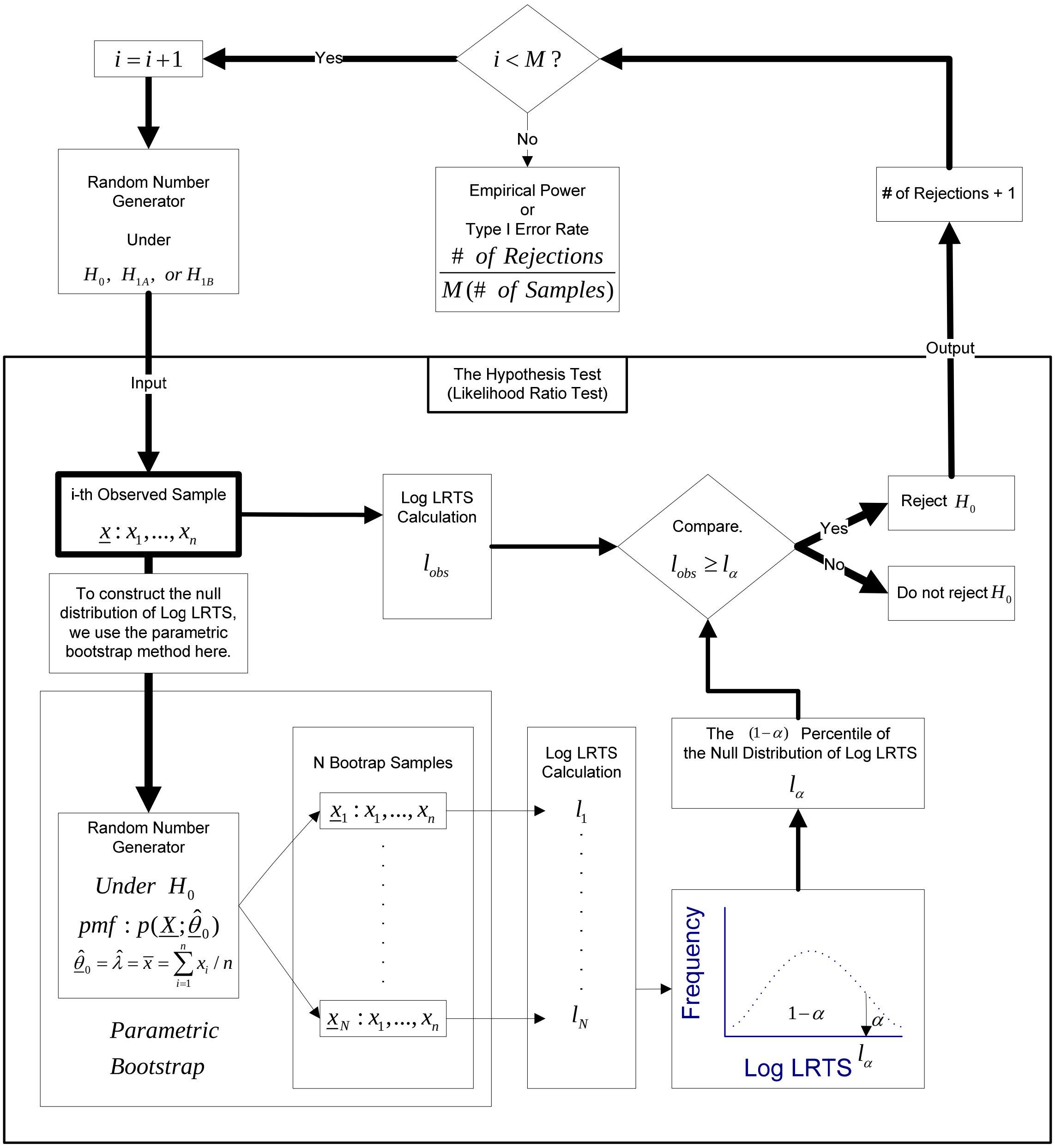
Step 4. Construct the empirical null distribution using these values of the log LRT statistics and find the $(1-\alpha)$ percentile of the empirical null distribution. This percentile value is denoted by l_α.

Step 5. Compare l_{obs} and l_α. If l_{obs} is bigger than l_α then we reject the null hypothesis.

Step 6. Count the number of times we reject the null hypothesis over M samples and divide it by the number of samples M. The calculated value is the empirical power rate of the test.

The steps to calculate the empirical power and type I error rate of the LRT using the parametric bootstrap method is summarized with Figure 3.2

Figure 3.2: The flow chart of the steps for obtaining the empirical power and type I error rate of the LRT using the parametric bootstrap method.



3.2 Posterior Predictive Check (PPC) Methods

3.2.1 Posterior Predictive Check (PPC) Methods

The Posterior Predictive Check (PPC) method was proposed by Rubin [21] to evaluate efficiently the fit of a model to the observed data. The method adopts the posterior predictive distribution to generate observable sample values based on the observed sample values. If the model fits the observed sample values, then the observable sample values from the posterior predictive distribution will look the same as the observed sample values. By comparing the observable sample values to the observed sample values, we can check the suitability of the model for the observed data.

The posterior predictive density is derived as follows. We first define the posterior predictive density as

$$p(\underline{X}^{rep} \mid \underline{x}) = \int p(\underline{X}^{rep}, \underline{\theta} \mid \underline{x}) d\underline{\theta}$$

$$= \int p(\underline{X}^{rep} \mid \underline{\theta}, \underline{x}) \cdot p(\underline{\theta} \mid \underline{x}) d\underline{\theta}$$

$$= \int p(\underline{X}^{rep} \mid \underline{\theta}) \cdot p(\underline{\theta} \mid \underline{x}) d\underline{\theta}$$ (Gelman, Carlin, Stern, and Rubin [8]).

Here $\underline{X}^{rep}$ is an observable random sample of size n.

Usually, the posterior predictive density is hard to derive directly. So we use an alternative method to estimate it. First, based on the observed sample values, we find the prior distribution, $p(\underline{\theta})$, and calculate posterior density, $p(\underline{\theta} \mid \underline{x})$. Second, we generate N simulations from the posterior density and draw one replicate $\underline{x}^{rep}$ from the predictive distribution of each simulated value of $\underline{\theta}$. Through these steps, we get N replicates from the joint distribution, $p(\underline{X}^{rep}, \underline{\theta} \mid \underline{x})$. These replicates are considered as observable sample values drawn from the posterior predictive distribution.

In the context of our problem, we assume that the null distribution is a single Poisson distribution. Thus, we use a gamma distribution as a conjugate prior distribution:

$$\lambda \sim Gamma(\alpha, \beta)$$

$$p(\lambda) = \frac{1}{\tau(\alpha)\beta^{\alpha}} \lambda^{\alpha-1} e^{-\frac{\lambda}{\beta}}.$$

The posterior of λ is determined based on this conjugate gamma prior distribution and the observed sample values, $\underline{x} = (x_1, \cdots, x_n)$ as follows:

$$\lambda \mid \underline{x} \sim Gamma(\alpha + \sum x_i, \frac{1}{(n+1/\beta)})$$

$$p(\lambda \mid \underline{x}) = \frac{(n+1/\beta)^{\alpha+\sum x_i}}{\tau(\alpha + \sum x_i)} \lambda^{\alpha+\sum x_i - 1} e^{-\lambda(n+1/\beta)}.$$

From this posterior density we generate N simulation values for the parameter, say λ_i, $i = 1, ..., N$ and draw one replicated sample value, $\underline{x}_i^{rep}$, from the Poisson distribution of each simulated value, λ_i. For each replicated sample value, $\underline{x}_i^{rep}$, we calculate our LRT statistic. These replicated LRT statistic values are used to construct a null distribution of our LRT statistic. We then determine the 95^{th} percentile of this distribution. The re-sampling procedure using the PPC method can be summarized with Figure 3.3.

We consider two parameter conditions for the gamma conjugate prior distribution of λ suggested by Kepner and Wackerly [12] and Viallefont et al. [25] to find parameter settings for the prior that maximize the power of the PPC method in assessing the LRT statistic. Usually, the posterior distribution of λ, $p(\lambda | x)$, is derived without determination of the predictive distribution of X, $p(X)$, because the posterior density is proportional to $p(\lambda) \cdot p(x | \lambda)$. Kepner and Wackerly [12], however, noted that the choice of the parameter values for the prior can influence the behavior of the marginal density of X, $p(X)$. The marginal density of X can be written as follows:

$$p(X = x) = \int_{\lambda=0}^{\lambda=\infty} p(X = x, \lambda) d\lambda$$

$$= \int_{\lambda=0}^{\lambda=\infty} p(\lambda) \cdot p(X = x | \lambda) d\lambda$$

$$= \frac{1}{x! \beta^{\alpha} \tau(\alpha)} \int_{\lambda=0}^{\lambda=\infty} \lambda^{\alpha - x - 1} e^{-\lambda(1+1/\beta)} d\lambda$$

$$= \frac{\tau(\alpha + x)}{x! \tau(\alpha)} \left(\frac{\beta}{1+\beta} \right)^{x} \left(\frac{1}{1+\beta} \right)^{\alpha}, \quad x = 0, 1, 2, \ldots$$

To study the behavior of this predictive distribution of X, they considered

$$\frac{P(X = x+1)}{P(X = x)} = \frac{(\alpha + x)\beta}{(x+1)(1+\beta)}, \quad x = 0, 1, 2, \ldots$$

As one can see, $P(X = x)$ is decreasing if $x > \alpha\beta - \beta - 1$. That is, if $\alpha\beta - \beta - 1 < 0$ then $P(X = x)$ becomes a monotonic decreasing function. To avoid this, the following condition is suggested:

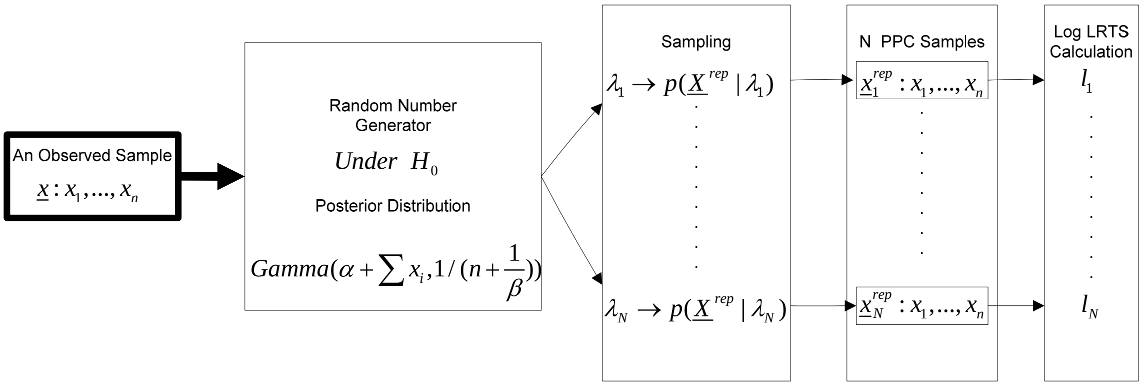
$$\alpha\beta - \beta - 1 > 0 \tag{3.3}$$

Viallefont et al. [25] suggested the following conditions based on their empirical studies:

$$\alpha = 1.1, \quad \alpha \cdot \beta = midrange = \frac{x_{(1)} + x_{(n)}}{2} \tag{3.4}$$

We consider 42 configurations of parameters for the prior under the condition (3.3) to study how the power and type I error rate of the PPC changes as α and β increase or decrease since the asymptotic distribution of the LRT statistic simulated by the PPC method may vary over the parameter values, α and β, for the prior of λ. The simulated results are compared to the power and type I error rates of the PPC generated under the condition (3.4) in the section 4.1.

Figure 3.3: The flow chart of the PPC method



3.2.2 Empirical Type I Error Rate of the PPC

The steps to calculate the empirical type I error rate of the hypothesis test A and B using the PPC method are as follows: (For the hypothesis test A, we follow Step 2A. For the hypothesis test B, we follow Step 2B.)

Step 1. Generate M samples of size n from a single Poisson distribution, under the null hypothesis.

Step 2A. For each observed sample, calculate the log LRT statistic, say l_{obs}, as (3.1).

Step 2B. For each observed sample, calculate the log LRT statistic, say l_{obs}, as (3.2).

Step 3. Collect N replicates, $\underline{x}_i^{rep}$, $i=1,...,N$, of the observed sample from the PPC re-sampling procedure and for each replicate calculate the log LRT statistic, say l_i, $i=1,...,N$.

Step 4. Construct the empirical null distribution using these values of the log LRT statistics and find the $(1-\alpha)$ percentile of the empirical null distribution. This percentile value is denoted by l_α.

Step 5. Compare l_{obs} and l_α. If l_{obs} is bigger than l_α then we reject the null hypothesis.

Step 6. Count the number of times we reject the null hypothesis over M samples and divide it by the number of samples M. The calculated value is the empirical type I error rate of the test.

3.2.3 Empirical Power of the PPC

The steps to calculate the empirical power of the hypothesis test A and B using the PPC method are as follows: (For the hypothesis test A, we follow Step 1A and 2A. For the hypothesis test B, we follow Step 1B and Step 2B.)

Step 1A. Generate M samples of size n from a mixture of two Poisson distributions, under H_{1A}.

Step 1B. Generate M samples of size n from a zero inflated Poisson distribution, under H_{1B}.

Step 2A. For each observed sample, calculate the log LRT statistic, say l_{obs}, as (3.1)

Step 2B. For each observed sample, calculate the log LRT statistic, say l_{obs}, as (3.2)

Step 3. Collect N replicates, $\underline{x}_i^{rep}$, $i = 1,...,N$, of the observed sample from the PPC re-sampling procedure and for each replicate calculate the log LRT statistic, say l_i, $i = 1,...,N$.

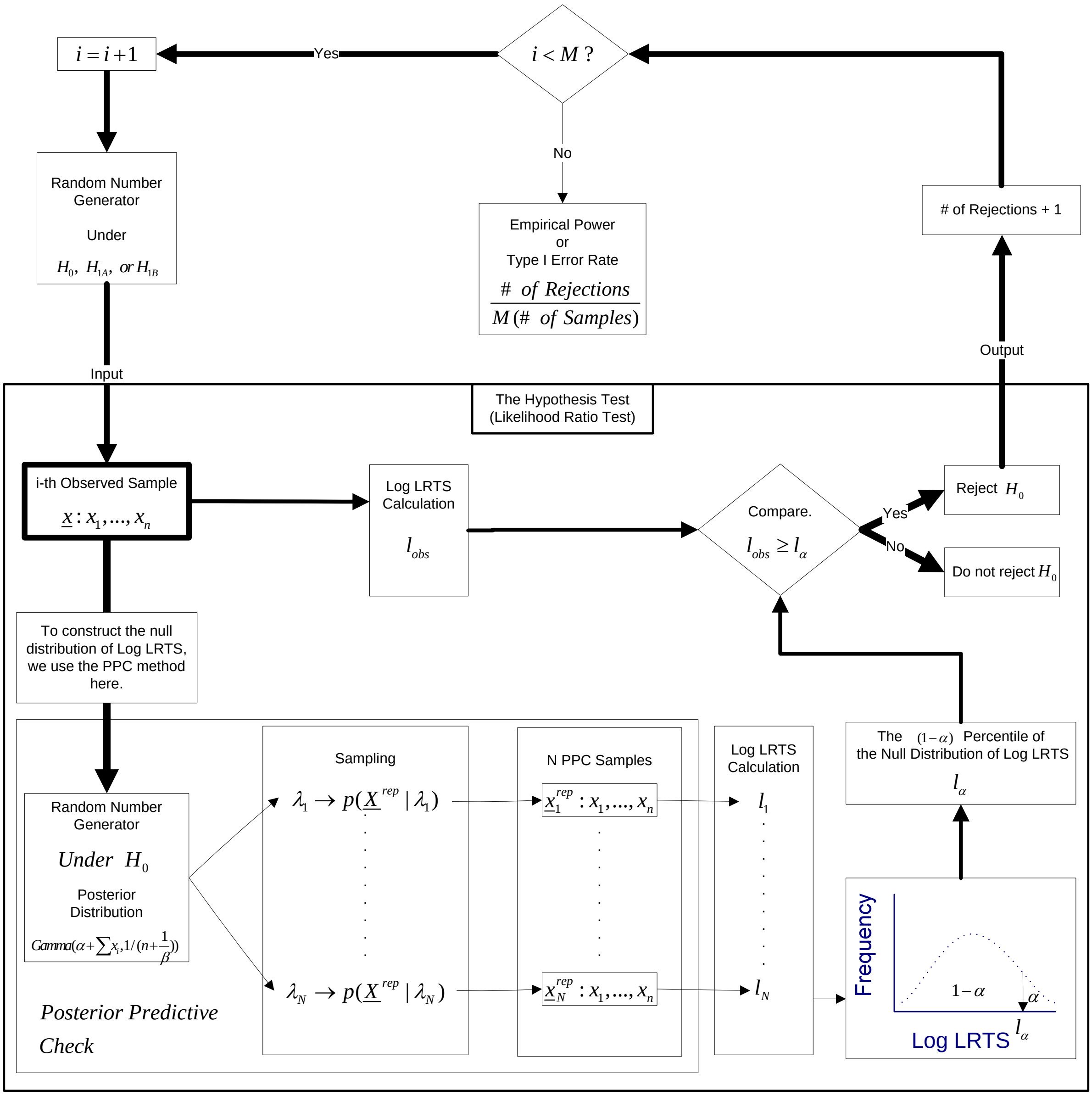
Step 4. Construct the empirical null distribution using these values of the log LRT statistics and find the $(1-\alpha)$ percentile of the empirical null distribution. This percentile value is denoted by l_α.

Step 5. Compare l_{obs} and l_α. If l_{obs} is bigger than l_α then we reject the null hypothesis.

Step 6. Count the number of times we reject the null hypothesis over M samples and divide it by the number of samples M. The calculated value is the empirical power of the test.

The steps to calculate the empirical power and type I error rate of the LRT using the PPC method is summarized with Figure 3.4.

Figure 3.4: The flow chart of the steps for obtaining the empirical power and type I error rate of the LRT using the PPC method.



3.3 Asymptotic Tests for Testing Zero Inflation in the Poisson Distribution

For the zero inflated Poisson alternative, the two likelihood ratio tests (LRT) using the bootstrap and PPC methods in assessing the statistical significance of the observed LRT statistic are compared to two asymptotic tests for discriminating the zero inflated Poisson distribution from the single Poisson distribution proposed by Rao and Chakravarti [20] and van den Broek [24].

3.3.1 Rao-Chakravarti Test

Rao and Chakravarti [20] proposed an asymptotic test for testing whether the number of zeros in the observed sample data is well-fitted with a single Poisson distribution in the case where there are no covariates. The test statistic was derived as follows based on the conditional probability of the observations, $x_1,\ x_2, \ldots,\ x_n$, from a single Poisson distribution given the total of the observations, $\mathrm{T} = x_1 + x_2 + \cdots + x_n = n\bar{x}$, which is multinomially-distributed:

$$\mathrm{U}=\frac{[n_0 - E(n_0)]}{\sqrt{\operatorname{var}(n_0)}} = \frac{n_0 - n(\frac{n-1}{n})^{n\bar{x}}}{\sqrt{n(\frac{n-1}{n})^{n\bar{x}} - n^2(\frac{n-1}{n})^{2n\bar{x}} + n(n-1)(\frac{n-2}{n})^{n\bar{x}}}} \sim N(0,1).$$

Here, n_0 is the number of zeros in the observed sample data, n is the sample size, and $\bar{x}$ is the mean of the observed sample data.

3.3.2 van den Broek Score Test

van den Broek [24] proposed a score test for testing a single Poisson distribution against a zero inflated Poisson distribution developed in the context of regression models which describe the relationship between covariate matrices and dependent variables. In the case where there are no covariates, his score test statistic reduces to the following statistic proposed by Cochran [2]:

$$S=\frac{(n_0 - ne^{-\bar{x}})^2}{ne^{-\bar{x}}(1-e^{-\bar{x}}) - ne^{-2\bar{x}}\bar{x}} \sim \chi_1^2,$$

where n_0 is the number of zeros in the observed sample data, n is the sample size, and $\bar{x}$ is the mean of the observed sample data.

Chapter 4

Simulations

4.1 Study of the Sensitivity of the Power of the Posterior Predictive Check (PPC) to the Choice of the Prior Parameters Values

In Section 3.2, we conjectured that the simulated power and type I error rate of the likelihood ratio test (LRT) using the PPC method as applied to estimate the null distribution of the LRT statistic would be sensitive to the choice of values for the parameters, α and β, of the prior distribution of λ. In this section, we conducted simulation studies to verify this conjecture and investigated whether there exists any significant trend in the power and type I error rates of the LRT using the PPC method as we change parameter values for the prior. The type I error and power rates of the PPC method were simulated based on the following parameter settings for the prior under the condition (3.3) suggested by Kepner and Wackerly [12]:

$$\begin{aligned} \alpha &= 1.1,\ 1.5,\ 2,\ 3,\ 4,\ 5,\ \textit{and}\ 10; \\ \beta &= 1/(\alpha-1)+t,\ t>0; \qquad (4.1) \\ t &= 0.1,\ 0.5,\ 1,\ 3,\ 5,\ \textit{and}\ 10. \end{aligned}$$

That is, 42 different configurations of parameters settings for the prior distribution of λ were considered.

The simulated results were compared to the empirical power and type I error rates of the

LRT using the PPC method calculated under the following conditions for the parameter values of the gamma prior suggested in Viallefont et al. [25]:

$$\alpha = 1.1, \quad \beta = \frac{midrange}{\alpha} = \frac{x_{(1)} + x_{(n)}}{2 \cdot \alpha}. \tag{4.2}$$

For this comparison study, 500 simulated samples of size 50, 100, and 200 were used. For each simulated sample, 500 replicates were generated through the posterior predictive check procedure to simulate the null distribution of the LRT statistic. The empirical power and type I error rates were calculated at significance level 0.05.

4.1.1 Simulated Type I Error Rate and Power of the LRT using the PPC Method for Testing Two-component Poisson Mixtures

We investigated if the choice of the parameter values for the prior influences the type I error rate of the LRT using the PPC method for testing two-component Poisson mixtures. We considered 4 configurations of the parameter settings for the null generating model. We simulated samples under the null hypothesis of the single Poisson distribution. We set λ = 1, 3, 5, and 10 and n = 50, 100, and 200. For each configuration of the parameter value and the sample size, 42 type I error rates of the LRT were calculated under the prior parameter condition (4.1). The results are shown in Table A.1 in Appendix A. Based on Table A.1, it appears that the type I error rate of the LRT using the PPC method is invariant to the parameter values, α and β, for the prior distribution of λ. There is no significant relationship between the type I error rate of the LRT and the parameter value of the null distribution. Table 4.1 presents summary statistics for the 42 simulated type I error rates of the LRT using the PPC method under the prior parameter setting (4.1) and the type I error rate of the LRT simulated under the prior parameter setting (4.2).

As one can see in Table 4.1, the type I error rates of the LRT using the PPC method simulated under both prior parameter conditions are about the same and appear to be close to the nominal significance level of 0.05.

Table 4.1: Summary statistics for the 42 type I error rates of the LRT using the PPC method simulated under the prior parameter values (4.1), compared to the type I error rate of the LRT simulated under the prior parameter condition (4.2) for testing two-component Poisson mixtures (H_{1A}) (N = 500 replicates per simulated sample)

		Kepner and Wackerly Values					Viallefont et al. Value
n	λ	Mean	SD	CV(%)	Min	Max	
50	1	0.042	0.004	9.9	0.04	0.05	0.04
	3	0.059	0.003	5.2	0.05	0.06	0.06
	5	0.068	0.004	6.7	0.06	0.08	0.07
	10	0.062	0.004	6.1	0.06	0.07	0.06
100	1	0.054	0.005	9.2	0.05	0.06	0.05
	3	0.051	0.003	6.4	0.05	0.06	0.05
	5	0.048	0.004	8.7	0.04	0.05	0.04
	10	0.052	0.005	9.5	0.04	0.06	0.05
200	1	0.056	0.005	8.7	0.05	0.06	0.06
	3	0.066	0.005	7.5	0.06	0.07	0.07
	5	0.058	0.004	7.2	0.05	0.06	0.06
	10	0.043	0.005	10.9	0.04	0.05	0.04

Notations: *n*: sample size; λ: mean of the null generating model (single Poisson); SD: standard deviation; CV: coefficient of variation.
Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 30 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05.

Secondly, we investigated whether the power of the LRT using the PPC method for testing the two-component Poisson alternative is sensitive to the choice of the prior parameters in the similar way. To generate samples under the alternative, we considered 20 different configurations of the parameter settings with π_1 = 0.1, 0.3, 0.5, 0.7 and 0.9 and (λ_1, λ_2) = (1, 4), (3, 6), (2, 4), and (3, 5) for sample sizes *n* = 50, 100, and 200. For each configuration of the parameter settings, we calculated 42 power rates of the LRT under the prior parameter condition (4.1). The results are shown in Table B.1 in Appendix B. From the results given in Table B.1, we concluded that there is no significant relationship between the power of the LRT using the PPC method for testing two-component Poisson mixtures and the parameter values, α and β, for the gamma conjugate prior of λ. Table 4.2 compares summary statistics for the 42 power rates of the LRT simulated under the condition (4.1) to the power rate of the LRT simulated under the condition (4.2). One can see that both parameter conditions for the prior result in almost equal power.

Table 4.2: Summary statistics for the 42 power rates of the LRT using the PPC method simulated under the prior parameter values (4.1), compared to the power rate of the LRT simulated under the prior parameter condition (4.2) for testing two-component Poisson mixtures (H_{1A}) (N = 500 replicates per simulated sample)

					Kepner and Wackerly Values					Viallefont et al. Value
π_1	n	λ_1	λ_2	D	Mean	SD	CV(%)	Min	Max	
0.1	50	1	4	3	0.357	0.008	2.1	0.34	0.37	0.35
		3	6	3	0.187	0.006	3.2	0.17	0.20	0.19
		2	4	2	0.134	0.005	3.7	0.13	0.14	0.14
		3	5	2	0.110	0.005	4.4	0.10	0.12	0.10
	100	1	4	3	0.520	0.007	1.3	0.50	0.53	0.52
		3	6	3	0.269	0.006	2.4	0.26	0.28	0.26
		2	4	2	0.185	0.005	2.7	0.18	0.19	0.18
		3	5	2	0.121	0.004	3.3	0.11	0.13	0.12
	200	1	4	3	0.801	0.004	0.5	0.79	0.81	0.80
		3	6	3	0.405	0.006	1.5	0.39	0.41	0.39
		2	4	2	0.220	0.006	2.7	0.20	0.23	0.21
		3	5	2	0.175	0.005	2.9	0.17	0.18	0.17
0.3	50	1	4	3	0.826	0.005	0.7	0.81	0.83	0.83
		3	6	3	0.470	0.007	1.4	0.46	0.48	0.46
		2	4	2	0.329	0.006	1.7	0.32	0.34	0.33
		3	5	2	0.232	0.006	2.6	0.22	0.24	0.24
	100	1	4	3	0.982	0.004	0.4	0.98	0.99	0.98
		3	6	3	0.710	0.006	0.9	0.70	0.72	0.71
		2	4	2	0.457	0.007	1.6	0.44	0.47	0.46
		3	5	2	0.295	0.006	1.9	0.28	0.30	0.30
	200	1	4	3	1.000	0	0	1.00	1.00	1.00
		3	6	3	0.922	0.004	0.4	0.92	0.93	0.93
		2	4	2	0.721	0.007	0.9	0.71	0.73	0.72
		3	5	2	0.473	0.007	1.5	0.46	0.49	0.48

Notations: π_1: mixing proportion of the first component; n: sample size; λ_1: the first component mean; λ_2: the second component mean; D: $\lambda_2 - \lambda_1$; SD: standard deviation; CV: coefficient of variation.
Notes: 1. 500 simulated samples of size 50, 100, and 200 and 10 random starting points were used. 2. The nominal significance level was 0.05.

Table 4.2 (Continued): Summary statistics for the 42 power rates of the LRT using the PPC method simulated under the prior parameter values (4.1), compared to the power rate of the LRT simulated under the prior parameter condition (4.2) for testing two-component Poisson mixtures (H_{1A}) (N = 500 replicates per simulated sample)

					Kepner and Wackerly Values					Viallefont et al. Value
π_1	n	λ_1	λ_2	D	Mean	SD	CV(%)	Min	Max	
0.5	50	1	4	3	0.958	0.004	0.5	0.95	0.96	0.96
		3	6	3	0.632	0.006	0.9	0.62	0.64	0.63
		2	4	2	0.403	0.006	1.5	0.39	0.41	0.40
		3	5	2	0.278	0.006	2.1	0.27	0.29	0.27
	100	1	4	3	1.000	0	0	1.00	1.00	1
		3	6	3	0.841	0.005	0.6	0.83	0.85	0.84
		2	4	2	0.595	0.006	0.9	0.59	0.61	0.59
		3	5	2	0.442	0.009	1.9	0.42	0.46	0.44
	200	1	4	3	1.000	0	0	1.00	1.00	1
		3	6	3	0.985	0.005	0.5	0.98	0.99	0.98
		2	4	2	0.871	0.005	0.6	0.86	0.88	0.87
		3	5	2	0.665	0.007	1.1	0.65	0.68	0.67
0.7	50	1	4	3	0.953	0.005	0.5	0.95	0.96	0.96
		3	6	3	0.613	0.007	1.1	0.60	0.63	0.61
		2	4	2	0.366	0.006	1.7	0.35	0.38	0.37
		3	5	2	0.256	0.007	2.7	0.24	0.27	0.26
	100	1	4	3	1.000	0	0	1.00	1.00	1
		3	6	3	0.790	0.006	0.8	0.78	0.80	0.79
		2	4	2	0.566	0.006	1.0	0.56	0.58	0.57
		3	5	2	0.402	0.006	1.5	0.39	0.41	0.39
	200	1	4	3	1.000	0	0	1.00	1.00	1
		3	6	3	0.980	0	0	0.98	0.98	0.98
		2	4	2	0.810	0.005	0.6	0.80	0.82	0.81
		3	5	2	0.627	0.006	1.0	0.62	0.64	0.63

Notations: π_1: mixing proportion of the first component; n: sample size; λ_1: the first component mean; λ_2: the second component mean; D: $\lambda_2 - \lambda_1$; SD: standard deviation; CV: coefficient of variation.
Notes: 1. 500 simulated samples of size 50, 100, and 200 and 10 random starting points were used. 2. The nominal significance level was 0.05.

Table 4.2 (Continued): Summary statistics for the 42 power rates of the LRT using the PPC method simulated under the prior parameter values (4.1), compared to the power rate of the LRT simulated under the prior parameter condition (4.2) for testing two-component Poisson mixtures (H_{1A}) (N = 500 replicates per simulated sample)

					Kepner and Wackerly Values					Viallefont et al. Value
π_1	n	λ_1	λ_2	D	Mean	SD	CV(%)	Min	Max	
0.9	50	1	4	3	0.619	0.006	1.0	0.61	0.63	0.62
		3	6	3	0.285	0.006	2.1	0.27	0.30	0.28
		2	4	2	0.179	0.006	3.2	0.17	0.19	0.18
		3	5	2	0.127	0.006	4.4	0.12	0.14	0.13
	100	1	4	3	0.885	0.006	0.7	0.88	0.90	0.89
		3	6	3	0.376	0.005	1.3	0.37	0.38	0.38
		2	4	2	0.286	0.005	1.7	0.28	0.29	0.28
		3	5	2	0.185	0.007	3.6	0.17	0.20	0.18
	200	1	4	3	0.990	0	0	0.99	0.99	0.99
		3	6	3	0.692	0.006	0.9	0.68	0.70	0.70
		2	4	2	0.429	0.006	1.3	0.42	0.44	0.43
		3	5	2	0.301	0.006	2.0	0.29	0.31	0.30

Notations: π_1: mixing proportion of the first component; n: sample size; λ_1: the first component mean; λ_2: the second component mean; D: $\lambda_2 - \lambda_1$; SD: standard deviation; CV: coefficient of variation.
Notes: 1. 500 simulated samples of size 50, 100, and 200 and 10 random starting points were used. 2. The nominal significance level was 0.05.

4.1.2 Simulated Type I Error Rate and Power of the LRT using the PPC Method for Testing ZIPs

For the zero inflated Poisson (ZIP) alternative, we investigated whether the choice of the prior parameter values impacts on the type I error rate of the LRT using the PPC method. To generate samples under the null hypothesis that the data came from a single Poisson distribution, we considered 3 configurations of the parameter settings with λ = 1, 2, and 3 for sample sizes n = 50, 100, and 200. For each configuration of the parameter settings, we calculated 42 type I error rates of the LRT under the prior parameter condition (4.1). The detailed results can be found in Table A.2 in Appendix A. From the results given in Table A.2, it seems that there is no significant relationship between the parameters for the prior and the type I error rate of the LRT using the PPC for testing ZIPs. Table 4.3 compares summary statistics for the 42 type I error rates of the LRT simulated under the condition (4.1) to the type I error rate of the LRT simulated under the condition (4.2). As you can see, both parameter conditions for the prior generate almost equal type I error rates, which are close to the nominal level of 0.05.

Table 4.3: Summary statistics for the 42 type I error rates of the LRT using the PPC method simulated under the prior parameter values (4.1), compared to the type I error rate of the LRT simulated under the prior parameter condition (4.2) for testing zero inflated Poisson distributions (H_{1B}) (N = 500 replicates per simulated sample)

		Kepner and Wackerly Values					Viallefont et al. Value
N	λ	Mean	SD	CV(%)	Min	Max	
50	1	0.049	0.005	9.7	0.04	0.06	0.05
	2	0.049	0.004	8.8	0.04	0.06	0.05
	3	0.066	0.005	7.4	0.06	0.07	0.07
100	1	0.051	0.004	6.9	0.05	0.06	0.06
	2	0.058	0.005	8.4	0.05	0.07	0.06
	3	0.057	0.005	9.1	0.05	0.07	0.06
200	1	0.046	0.005	10.8	0.04	0.05	0.04
	2	0.058	0.004	6.5	0.05	0.06	0.06
	3	0.054	0.005	9.2	0.05	0.06	0.06

Notations: *n*: sample size; λ: mean of the null generating model (single Poisson); SD: standard deviation; CV: coefficient of variation.
Notes: 1. 500 simulated samples of size 50, 100, and 200 and 30 random starting points were used. 2. The nominal significance level was 0.05.

In the same manner, we studied the sensitivity of the power of the LRT using the PPC method for testing the zero inflation Poisson (ZIP) distribution to the choice of the prior parameters. To generate samples under the alternative, we considered 9 different configurations of the parameter settings with π_1 = 0.1, 0.3, and 0.5 and λ_2 = 1, 2, and 3 for sample sizes *n* = 50, 100, and 200. For each configuration of the parameter settings, 42 power rates of the LRT were calculated under the prior parameter condition (4.1). The results are shown in Table B.2 in Appendix B. One can see in Table B.2 that there is no significant relationship between the power of the LRT using the PPC method for testing ZIPs and the parameter values, α and β, for the gamma conjugate prior of λ. Table 4.4 reports summary statistics for the 42 power rates of the LRT simulated under the condition (4.1) and the power of the LRT simulated under the condition (4.2). As one can see, both parameter conditions for the prior result in almost equal powers.

Table 4.4: Summary statistics for the 42 power rates of the LRT using the PPC method simulated under the prior parameter values (4.1), compared to the power rate of the LRT simulated under the prior parameter condition (4.2) for testing zero inflated Poisson distributions (H_{1B}) (N = 500 replicates per simulated sample)

			Kepner and Wackerly Values					Viallefont et al. Value
π_1	n	λ_2	Mean	SD	CV(%)	Min	Max	
0.1	50	1	0.145	0.006	3.8	0.14	0.16	0.15
		2	0.401	0.007	1.8	0.38	0.41	0.40
		3	0.735	0.005	0.7	0.73	0.74	0.73
	100	1	0.232	0.008	3.4	0.22	0.25	0.24
		2	0.606	0.007	1.1	0.59	0.62	0.61
		3	0.921	0.003	0.4	0.91	0.93	0.92
	200	1	0.304	0.007	2.2	0.29	0.32	0.29
		2	0.831	0.005	0.6	0.82	0.84	0.84
		3	0.99	0	0	0.99	0.99	0.99
0.3	50	1	0.427	0.007	1.6	0.42	0.44	0.43
		2	0.937	0.005	0.5	0.93	0.94	0.93
		3	1.000	0	0	1.00	1.00	1.00
	100	1	0.678	0.006	0.9	0.67	0.69	0.68
		2	0.997	0.005	0.5	0.99	1.00	0.99
		3	1.000	0	0	1.00	1.00	1.00
	200	1	0.928	0.005	0.5	0.92	0.94	0.93
		2	1.000	0	0	1.00	1.00	1.00
		3	1.000	0	0	1.00	1.00	1.00
0.5	50	1	0.715	0.007	1.0	0.70	0.73	0.71
		2	1.000	0	0	1.00	1.00	1.00
		3	1.000	0	0	1.00	1.00	1.00
	100	1	0.921	0.004	0.4	0.91	0.93	0.92
		2	1.000	0	0	1.00	1.00	1.00
		3	1.000	0	0	1.00	1.00	1.00
	200	1	1.000	0	0	1.00	1.00	1.00
		2	1.000	0	0	1.00	1.00	1.00
		3	1.000	0	0	1.00	1.00	1.00

Notations: π_1: mixing proportion of the first component; n: sample size; λ_2: the second component mean; SD: standard deviation; CV: coefficient of variation.

Notes: 1. 500 simulated samples of size 50, 100, and 200 and 30 random starting points were used. 2. The nominal significance level was 0.05.

4.1.3 Using the Prior Parameter Condition Proposed by Viallefont et al. in the Power Comparison Study.

Based on the simulation results given in Sections 4.1.1 and 4.1.2, we concluded that the type I error rate and power of the LRT using the PPC method in assessing the LRT statistic for testing each of the alternatives are invariant to the choice of the parameter values for the prior distribution. As well, we noted that both parameter conditions, (4.1) and (4.2), suggested by Kepner and Wackerly [12] and Viallefont et al. [25] result in essentially equal powers. Therefore, we decided to use the prior parameter conditions (4.2) suggested by Viallefont et al. [25] in our power studies.

4.2 Power Comparison of Likelihood Ratio Tests for Two-component Poisson Mixtures (H_{1A})

We first compared the type I error rate of the LRT using the PPC method for testing the two-component Poisson mixture with the LRT using the bootstrap method to see whether the two LRTs gives comparable type I error rates, which are close to the nominal level of 0.05. To simulate samples under the null hypothesis that the data came from a single Poisson distribution, we considered 4 configurations of the parameter settings with $\lambda = 1, 3, 5$ and 10 for sample sizes $n = 50, 100$, and 200. For each configuration, we simulated 500 samples. For each simulated sample, we replicated 500 PPC samples and 500 bootstrap samples under the null hypothesis. For each replicated sample, we calculated the LRT statistic to estimate the null distribution of the LRT statistic. The empirical type I error rate of each LRT was calculated based on the 95th percentile of the simulated null distribution of the LRT statistic. To determine the MLEs of the parameters of the two-component Poisson mixture alternative, 30 random starting points were used for the EM algorithm shown in Table 2.1. From the results shown in Table 4.5, one can see that both methods resulted in almost equal type I error rates, which appear to be close to the nominal type I error of 0.05. The type I error rate of the LRT seemed to be invariant to the value of the mean of the null generating model.

Table 4.5: Simulated type I error rate of the LRT for testing two-component Poisson mixtures (H_{1A}): comparison of the PPC and bootstrap methods

Simulated Type I Error Rate of the LRT for Testing Two-component Poisson Mixtures (N = 500 replicates per simulated sample)

	n = 50		n = 100		n = 200	
λ	PPC	BS	PPC	BS	PPC	BS
1	0.04	0.04	0.05	0.06	0.06	0.06
3	0.06	0.06	0.05	0.06	0.07	0.07
5	0.07	0.07	0.04	0.05	0.06	0.06
10	0.06	0.06	0.05	0.05	0.04	0.05

Notations: n: sample size; λ: mean of the null generating model (single Poisson); PPC: the LRT using the posterior predictive check method; BS: the LRT using the parametric bootstrap method.
Notes: 1. 500 simulated samples of size 50, 100, and 200 and 30 random starting points were used. 2. The nominal significance level was 0.05. 3. The margin of error at 95% confidence is ±0.02 for each configuration.

Next, we compared the power of the LRT using the PPC method with the LRT using the bootstrap method. To simulate samples under the alternative hypothesis that the data came from a mixture of two Poisson distributions, we considered 20 configurations of the parameter settings with π_1 = 0.1, 0.3, 0.5, 0.7 and 0.9 and (λ_1, λ_2) = (1, 4), (3, 6), (2, 4), and (3, 5) for sample sizes n = 50, 100, and 200. For each configuration, we simulated 500 samples. For each simulated sample, we replicated 500 PPC samples and 500 bootstrap samples under the null hypothesis. For each replicated sample, we calculated the LRT statistic to estimate the null distribution of the LRT statistic. The power of each LRT was calculated based on the 95th percentile of the simulated null distribution of the LRT statistic. To determine the MLEs of the parameters of the two-component Poisson mixture alternative, we used 10 random starting points for the EM algorithm shown in Table 2.1. In addition, we applied McNemar's test with the significance level of 0.05 and 0.01 to determine whether the two LRTs result in significantly different powers.

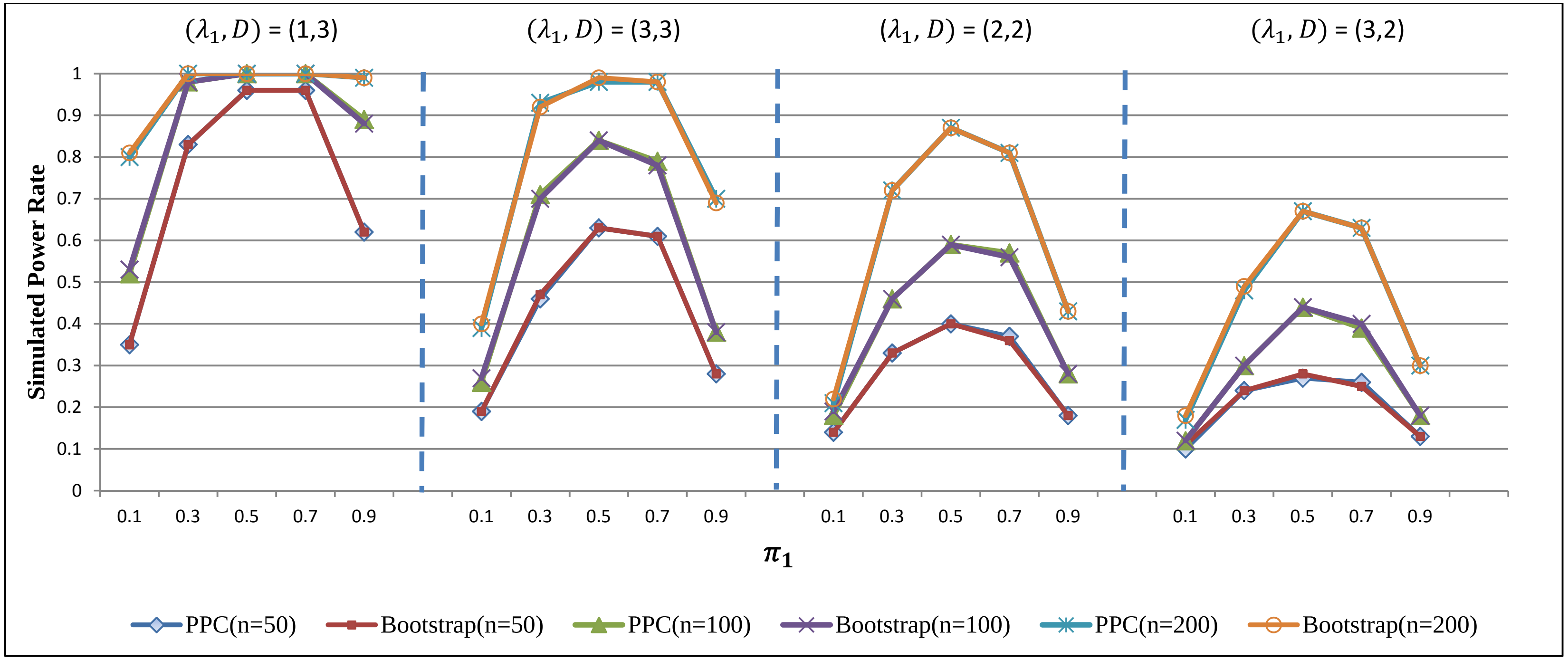
Table 4.6: Simulated power of the LRT for testing two-component Poisson mixtures (H_{1A}): comparison of the PPC and bootstrap methods

				Power of the LRT for Testing Two-component Poisson Mixtures (N = 500 replicates per simulated sample)					
				n = 50		n = 100		n = 200	
π_1	λ_1	λ_2	D	PPC	BS	PPC	BS	PPC	BS
0.1	1	4	3	0.35	0.35	0.52	0.53	0.80	0.81
	3	6	3	0.19	0.19	0.26	0.27	0.39	0.40
	2	4	2	0.14	0.14	0.18	0.19	0.21	0.22
	3	5	2	0.10	0.11	0.12	0.12	0.17	0.18
0.3	1	4	3	0.83	0.83	0.98	0.98	1.00	1.00
	3	6	3	0.46	0.47	0.71	0.70	0.93	0.92
	2	4	2	0.33	0.33	0.46	0.46	0.72	0.72
	3	5	2	0.24	0.24	0.30	0.30	0.48	0.49
0.5	1	4	3	0.96	0.96	1.00	1.00	1.00	1.00
	3	6	3	0.63	0.63	0.84	0.84	0.98	0.99
	2	4	2	0.40	0.40	0.59	0.59	0.87	0.87
	3	5	2	0.27	0.28	0.44	0.44	0.67	0.67
0.7	1	4	3	0.96	0.96	1.00	1.00	1.00	1.00
	3	6	3	0.61	0.61	0.79	0.78	0.98	0.98
	2	4	2	0.37	0.36	0.57	0.56	0.81	0.81
	3	5	2	0.26	0.25*	0.39	0.40	0.63	0.63
0.9	1	4	3	0.62	0.62	0.89	0.88	0.99	0.99
	3	6	3	0.28	0.28	0.38	0.38	0.70	0.69
	2	4	2	0.18	0.18	0.28	0.28	0.43	0.43
	3	5	2	0.13	0.13	0.18	0.18	0.30	0.29

Notations: π_1: mixing proportion of the first component; *n*: sample size; λ_1: the first component mean; λ_2: the second component mean; *D*: $\lambda_2 - \lambda_1$; PPC: the LRT using the posterior predictive check method; BS: the LRT using the parametric bootstrap method.
Notes: 1. 500 simulated samples of size 50, 100, and 200 and 10 random starting points were used. 2. The nominal significance level was 0.05. 3. The margin of error at 95% confidence is ±0.04 for each configuration. 4. Significantly different in power compared to the LRT using the PPC method (McNemar's Test, * 0.05; ** 0.01)

The simulated power values of both methods shown in Table 4.6 are illustrated in Figure 4.1. As can be better seen from the figure, both methods generated almost equal power for each configuration. As expected, the power of the LRT increased as the sample size *n* increases, the mixing proportion of the first Poisson component π_1 becomes closer to 0.5, or the difference between two Poisson component means, *D*, increases. Given equal values of *D* and π_1, the power of the LRT increased as the mean of the first Poisson component decreases.

Figure 4.1: Comparison of the simulated Power of the LRT using the PPC and bootstrap methods for testing two-component Poisson mixtures (H_{1A})



Notations: π_1: mixing proportion of the first component; *n*: sample size; λ_1: the first component mean; λ_2: the second component mean; *D*: $\lambda_2 - \lambda_1$.
Notes: 1. 500 simulated samples of size 50, 100, and 200 and 10 random starting points were used. 2. *N* = 500 replicates per simulated sample. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.04 for each configuration.

4.3 Power Comparison of Tests for Zero Inflation in the Poisson Distribution (H_{1B})

For the zero inflated Poisson alternative, we compared the type I error rate of the LRT using the PPC method to that of the LRT using the bootstrap method and two asymptotic tests proposed by Rao and Chakravarti [20] and van den Broek [24]. To simulate samples under the null hypothesis that the sample data came from a single Poisson distribution, we considered 3 configurations of parameters with λ = 1, 2, and 3 for sample sizes n = 50, 100, and 200. For each configuration, we simulated 500 samples. For each simulated sample, we calculated the LRT statistic, Rao-Chakravarti test statistic, and van den Broek test statistic. The decision to reject the null hypothesis for each observed Rao-Chakravarti test statistic was based on the 95th percentile value, 1.96, of the standard normal distribution. The decision to reject the null hypothesis for each observed van den Broek test statistic was based on the 95th percentile value, 3.84, of the chi-squared distribution with 1 degree of freedom. This decision for each observed LRT statistic was based on the 95th percentile value of the null distribution of the LRT statistic simulated by the PPC and bootstrap methods. To simulate the null distribution of the LRT statistic, we replicated 500 PPC samples and 500 bootstrap samples on the basis of the simulated sample and for each replicated PPC and bootstrap sample calculated the LRT statistic. To determine the MLEs of the parameters of the zero inflated Poisson distribution, we used 30 random starting points for the EM algorithm shown in Table 2.2.

As one can see in Table 4.7, it appears that the LRTs using the PPC and bootstrap methods, Rao-Chakravarti test, and van den Broek test result in comparable type I error rates, which are close to the nominal type I error of 0.05.

Table 4.7: Simulated type I error rate of the LRT using the PPC, compared to that of the LRT using the bootstrap, Rao-Chakravarti test, and van den Broek score test for testing ZIP s(H_{1B})

Simulated Type I Error Rate of the Tests for Detecting Zero Inflation in a Poisson
(N = 500 replicates per simulated sample)

	n = 50				n = 100				n = 200			
λ	PPC	BS	RC	SC	PPC	BS	RC	SC	PPC	BS	RC	SC
1	0.05	0.05	0.05	0.05	0.06	0.05	0.05	0.04	0.04	0.04	0.04	0.05
2	0.05	0.05	0.05	0.04	0.06	0.06	0.06	0.05	0.06	0.06	0.06	0.06
3	0.07	0.07	0.08	0.04	0.06	0.05	0.07	0.04	0.06	0.05	0.06	0.05

Notations: n: sample size; λ: mean of the null generating model (single Poisson); PPC: the LRT using the posterior predictive check method; BS: the LRT using the parametric bootstrap method; RC: Rao and Chakravarti criterion; SC: van den Broek score test.
Notes: 1. 500 simulated samples of size 50, 100, and 200 and 30 random starting points were used. 2. The nominal significance level was 0.05. 3. The margin of error at 95% confidence is ±0.02 for each configuration.

Next, we compared the power of the LRT using the PPC method with the LRT using the bootstrap method and two asymptotic tests proposed by Rao and Chakravarti [20] and van den Broek [24] for testing zero inflation in a Poisson distribution. To simulate samples under the alternative hypothesis that the data came from a zero inflated Poisson distribution, we considered 9 configurations of the parameter settings with π_1 = 0.1, 0.3, and 0.5 and λ_2 = 1, 2, and 3 for sample sizes n = 50, 100, and 200. For each configuration, we simulated 500 samples. For each simulated sample, we calculated the LRT statistic, Rao-Chakravarti test statistic, and van den Broek test statistic. The decision to reject the null hypothesis for each observed Rao-Chakravarti test statistic was based on the 95th percentile value, 1.96, of the standard normal distribution. The decision to reject the null hypothesis for each observed van den Broek test statistic was based on the 95th percentile value, 3.84, of the chi-squared distribution with 1 degree of freedom. This decision for each observed LRT statistic was based on the 95th percentile value of the null distribution of the LRT statistic simulated by the PPC and bootstrap methods. To simulate the null distribution of the LRT statistic, we replicated 500 PPC samples and 500 bootstrap samples on the basis of the simulated sample and for each replicated PPC and bootstrap sample calculated the LRT statistic. To determine the MLEs of the parameters of the zero inflated Poisson distribution, we used 10 random starting points for the EM algorithm shown in Table 2.2. In addition, for each configuration, we conducted McNemar's test to see whether the power of the LRT using the PPC method is significantly different from that of the LRT using the bootstrap, the Rao-Chakravarti test and the van den Broek score test with the significance levels of 0.05 and 0.01. The simulated power of each test for each configuration is shown in Table 4.8. The table also contains the results of the McNemar's Test.

Figure 4.2 illustrates the simulation results shown in Table 4.8. As one can see in the figure, the two LRTs using the PPC and bootstrap methods and the Rao-Chakravarti test result in comparable powers for each configuration, which are significantly higher than the power of the van den Broek score test. As expected, the power of each test increased as the sample size n increases, the mixing proportion, π_1, of the first Poisson component with zero mean gets closer to 0.5, or the mean of the second Poisson component increases.

Table 4.8: Simulated power of the LRT using the PPC , compared to that of the LRT using the bootstrap, Rao-Chakravarti test, and van den Broek score test for testing ZIPs (H_{1B})

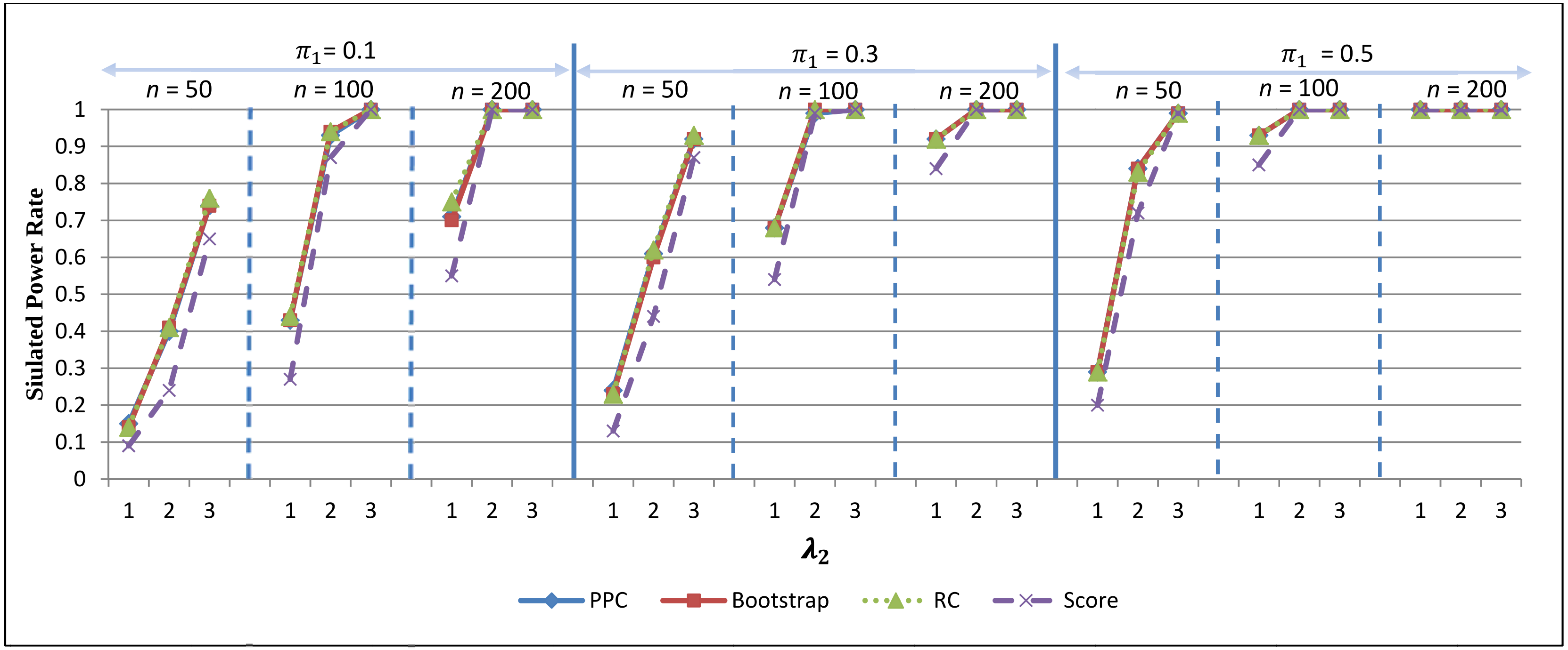
Simulated Power of the Tests for Detecting Zero Inflation in a Poisson
(*N* = 500 replicates per simulated sample)

		n = 50				*n* = 100				*n* = 200			
π_1	λ_2	PPC	BS	RC	SC	PPC	BS	RC	SC	PPC	BS	RC	SC
0.1	1	0.15	0.14	0.14	0.09**	0.24	0.23	0.23	0.13**	0.29	0.29	0.29	0.20**
	2	0.40	0.41	0.41	0.24**	0.61	0.6	0.62	0.44**	0.84	0.84	0.83	0.72**
	3	0.74	0.74	0.76**	0.65**	0.92	0.92	0.93	0.87**	0.99	0.99	0.99	0.99
0.3	1	0.43	0.43	0.44	0.27**	0.68	0.68	0.68	0.54**	0.93	0.93	0.93	0.85**
	2	0.93	0.94	0.94	0.87**	0.99	1.00	1.00	0.99	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.5	1	0.71	0.70	0.75**	0.55**	0.92	0.92	0.92	0.84**	1.00	1.00	1.00	1.00
	2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Notations: π_1: mixing proportion of the first component; *n*: sample size; λ_2: the second component mean; PPC: the LRT using the posterior predictive check method; BS: the LRT using the parametric bootstrap method; RC: Rao and Chakravarti criterion; SC: van den Broek score test

Notes: 1. 500 simulated samples of size 50, 100, and 200 and 10 random starting points were used. 2. The nominal significance level was 0.05. 3. The margin of error at 95% confidence is ±0.04 for each configuration. 4. Significantly different in power compared to the LRT using the PPC method (McNemar's Test, * 0.05; ** 0.01)

Figure 4.2: Comparison of the simulated power of the LRT using the PPC and bootstrap methods, Rao-Chakravarti test, and van den Broek score test for testing ZIPs (H_{1B})



Notations: π_1: mixing proportion of the first component; *n*: sample size; λ_2: the second component mean; PPC: the LRT using the posterior predictive check method; Bootstrap: the LRT using the parametric bootstrap method; RC: Rao and Chakravarti criterion; Score: van den Broek score test

Notes: 1. 500 simulated samples of size 50, 100, and 200 and 10 random starting points were used. 2. N = 500 replicates per simulated sample. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.04 for each configuration.

Chapter 5

Summary and Conclusions

It is well known that the regularity conditions for the classical theorem for the null distribution of the likelihood ratio test (LRT) statistic break down in a test where the alternative hypothesis is a finite mixture model. An alternative approach broadly used to resolve this issue is using the parametric bootstrap method in estimating the P-value of the observed LRT statistic (McLachlan [15]).

In this study, we demonstrated that a resampling procedure adopted in a Bayesian model checking procedure, called posterior predictive check (PPC), provides power comparable to the bootstrap method in assessing the statistical significance of the observed LRT statistic. For this comparison study, we considered two hypothesis tests where we test the null hypothesis that the sample data arose from a single Poisson distribution against (1) the alternative hypothesis that the sample data arose from a two-component Poisson mixture distribution and (2) the alternative hypothesis that the data arose from a zero inflated Poisson distribution. For the first alternative hypothesis, we compared the power of the PPC method with the bootstrap method. For the second alternative hypothesis, we compared the power of the PPC method to that of the bootstrap method, Rao and Chakravarti [20] test, and van den Broek [24] score test. The comparison studies were conducted across various parameter settings and simulated data sets for each of these alternatives.

Before conducting our comparison studies, we investigated the sensitivity of the power and type I error rate of the LRT using the PPC method to the choice of parameter values for the prior distribution of λ. We conducted simulation studies for each of alternatives based on two parameter conditions for the prior distribution suggested in Kepner and Wackerly [12] and Viallefont et al. [25]. From the simulation results, we found that the power and type I error rates of the two hypothesis tests using the PPC method in assessing the observed LRT statistic are insensitive to the choice of parameter values for the prior distribution and the two parameter conditions result in similar power and type I error rates. Thus, for our power analyses, we concluded to use the parameter conditions suggested by Viallefont et al. [25].

The simulation results of our comparison studies showed that for the two-component Poisson mixture alternative the two LRTs using the PPC and bootstrap methods in estimating the null distribution of the LRT statistic generate equal type I error rates and powers. The type I error rates of both methods seemed to be close to the nominal significance level of 0.05. It appeared that the type I error rate is not associated with the mean value of the null generating model. As we expected, the power of the LRT using the PPC method for testing the two-component Poisson mixture alternative improved as the mixing proportion of the first component gets closer to 0.5, the difference between the two Poisson component means increases, or the sample size increases. For fixed values of the difference between the two Poisson component means and the mixing proportion of the first Poisson component, as we decrease the mean of the first component, the power of the LRT improved.

In the case of the zero inflated Poisson alternative, the two LRTs using the PPC and bootstrap methods, Rao-Chakravarti test, and van den Broek score test gave comparable type I error rates, which are very close to the nominal level of 0.05. The power of two LRTs and Rao-Chakravarti test were about the same but significantly greater than that of van den Broek score test. Generally the LRT using the PPC or bootstrap method in estimating the null distribution of the LRT statistic takes a much greater computation time than the two asymptotic tests since the LRT involves running EM algorithm to obtain MLEs of the alternative model. Thus, we recommend that one uses the Rao-Chakravarti criterion in testing a single Poisson distribution against a zero inflated Poisson distribution. The power of the four tests increased as expected, as the sample size increases, the mixing proportion of zeros becomes closer to 0.5, or the mean of the second component increases.

Our comparison study results calculated under the two-component Poisson mixture alternative are consistent with the simulation result shown in Lo [13] that the two LRT using the PPC and bootstrap methods generate comparable powers in testing the number of components in a normal mixture model with unequal variance. In the context of the zero inflated Poisson alternative, El-Shaarawi [7] showed that the LRT using the classical asymptotic null distribution of the LRT statistic in assessing the observed LRT statistic generates much lower power than the two asymptotic tests. However, the asymptotic theorem for the LRT statistic cannot be used for testing the zero inflated Poisson alternative since the zero inflated Poisson can be viewed as the two-component Poisson mixture where the first component mean is equal to zero. The theorem does not apply when the alternative is a finite mixture. Therefore, our simulation study results generated under the ZIP alternative are preferable.

In this dissertation, we only considered the simplest hypothesis test: a single Poisson distribution (k=1) versus a mixture of two Poisson distributions and a zero inflated Poisson distribution (k=2). Thus, the conclusion we made from the study of the sensitivity of the power and type I error rate of the LRT using the PPC method to the choice of parameter values for the prior distribution might not be true in testing the three- or four-component Poisson mixture alternative. A possible extension of this study for the future work would be to (1) investigate whether the choice of parameter values for the prior distribution is associated with the power of the LRT using the PPC method for testing the *k*-component Poisson mixture alternative, (2) find parameter settings for the prior distribution that maximize the power of the LRT using the PPC in assessing the observed LRT statistic, and (3) compare the power of the LRT based on the prior parameter setting with the LRT using the bootstrap method.

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Appendices

Appendix A: Simulated type I error rate of the LRT using the PPC method

Table A.1 – Simulated type I error rate of the LRT for testing two-component Poisson mixtures using the PPC method based 42 configurations of prior parameter values (4.1) (*N*= 500 replicates)

			Simulated Type I Error Rate (Kepner and Wackerly Values)					
			T					
n	λ	α	0.1	0.5	1	3	5	10
50	1	1.1	0.05	0.04	0.05	0.05	0.04	0.04
		1.5	0.05	0.04	0.04	0.05	0.04	0.04
		2	0.04	0.04	0.04	0.04	0.05	0.05
		3	0.04	0.05	0.04	0.04	0.04	0.04
		4	0.04	0.04	0.04	0.04	0.04	0.04
		5	0.04	0.04	0.04	0.04	0.04	0.04
		10	0.04	0.04	0.05	0.04	0.04	0.04
	3	1.1	0.06	0.06	0.06	0.06	0.06	0.06
		1.5	0.06	0.06	0.06	0.06	0.06	0.06
		2	0.06	0.06	0.06	0.05	0.06	0.05
		3	0.07	0.06	0.06	0.06	0.06	0.06
		4	0.06	0.06	0.06	0.06	0.06	0.06
		5	0.06	0.06	0.05	0.06	0.06	0.06
		10	0.06	0.06	0.06	0.06	0.06	0.06
	5	1.1	0.07	0.06	0.07	0.07	0.07	0.07
		1.5	0.06	0.07	0.07	0.07	0.07	0.07
		2	0.07	0.07	0.07	0.07	0.07	0.07
		3	0.07	0.07	0.07	0.06	0.06	0.07
		4	0.07	0.08	0.06	0.06	0.07	0.06
		5	0.07	0.07	0.06	0.07	0.07	0.07
		10	0.07	0.07	0.06	0.07	0.07	0.07
	10	1.1	0.06	0.06	0.06	0.06	0.07	0.06
		1.5	0.06	0.06	0.06	0.06	0.06	0.06
		2	0.07	0.07	0.07	0.06	0.06	0.06
		3	0.06	0.06	0.06	0.06	0.06	0.06
		4	0.06	0.06	0.06	0.06	0.06	0.06
		5	0.06	0.07	0.06	0.06	0.06	0.06
		10	0.06	0.06	0.07	0.06	0.06	0.07

Notations: *n*: sample size; λ: mean of the null generating model (single Poisson).
Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 30 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.02 for each configuration.

Table A.1 (Continued) – Simulated type I error rate of the LRT for testing two-component Poisson mixtures using the PPC method based 42 configurations of prior parameter values (4.1) (*N*= 500 replicates)

			Simulated Type I Error Rate (Kepner and Wackerly Values)					
			T					
n	λ	α	0.1	0.5	1	3	5	10
100	1	1.1	0.05	0.06	0.05	0.06	0.06	0.05
		1.5	0.05	0.05	0.06	0.05	0.06	0.06
		2	0.06	0.05	0.06	0.05	0.06	0.05
		3	0.05	0.05	0.05	0.06	0.05	0.06
		4	0.05	0.05	0.05	0.06	0.05	0.05
		5	0.05	0.05	0.06	0.06	0.06	0.05
		10	0.05	0.05	0.05	0.05	0.06	0.06
	3	1.1	0.05	0.05	0.05	0.05	0.05	0.05
		1.5	0.05	0.05	0.05	0.05	0.05	0.05
		2	0.06	0.05	0.05	0.05	0.05	0.05
		3	0.05	0.05	0.05	0.06	0.05	0.05
		4	0.05	0.05	0.05	0.05	0.06	0.05
		5	0.05	0.05	0.05	0.05	0.05	0.06
		10	0.06	0.05	0.05	0.05	0.05	0.05
	5	1.1	0.05	0.04	0.05	0.05	0.05	0.05
		1.5	0.05	0.04	0.05	0.05	0.05	0.05
		2	0.05	0.04	0.05	0.05	0.04	0.05
		3	0.04	0.05	0.05	0.05	0.05	0.05
		4	0.05	0.05	0.05	0.05	0.04	0.05
		5	0.04	0.05	0.05	0.05	0.05	0.04
		10	0.05	0.05	0.05	0.05	0.05	0.04
	10	1.1	0.05	0.06	0.05	0.05	0.05	0.06
		1.5	0.06	0.06	0.05	0.06	0.05	0.06
		2	0.05	0.05	0.05	0.06	0.05	0.05
		3	0.05	0.05	0.05	0.05	0.05	0.05
		4	0.06	0.05	0.05	0.05	0.05	0.05
		5	0.04	0.05	0.05	0.05	0.05	0.05
		10	0.06	0.04	0.05	0.05	0.05	0.05

Notations: *n*: sample size; λ: mean of the null generating model (single Poisson).
Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 30 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.02 for each configuration.

Table A.1 (Continued) – Simulated type I error rate of the LRT for testing two-component Poisson mixtures using the PPC method based 42 configurations of prior parameter values (4.1) (*N*= 500 replicates)

			Simulated Type I Error Rate (Kepner and Wackerly Values) *T*					
n	λ	α	0.1	0.5	1	3	5	10
200	1	1.1	0.06	0.06	0.06	0.06	0.05	0.06
		1.5	0.06	0.05	0.06	0.06	0.06	0.05
		2	0.05	0.06	0.06	0.05	0.06	0.06
		3	0.05	0.06	0.05	0.06	0.05	0.05
		4	0.06	0.06	0.06	0.06	0.06	0.05
		5	0.06	0.05	0.06	0.05	0.06	0.06
		10	0.05	0.05	0.05	0.05	0.06	0.06
	3	1.1	0.06	0.07	0.07	0.07	0.07	0.07
		1.5	0.07	0.07	0.06	0.07	0.06	0.07
		2	0.06	0.07	0.06	0.07	0.07	0.07
		3	0.07	0.07	0.06	0.07	0.06	0.07
		4	0.06	0.07	0.06	0.06	0.06	0.06
		5	0.06	0.06	0.06	0.07	0.06	0.07
		10	0.07	0.07	0.07	0.06	0.07	0.07
	5	1.1	0.05	0.06	0.06	0.05	0.05	0.06
		1.5	0.06	0.06	0.06	0.06	0.06	0.06
		2	0.06	0.05	0.06	0.05	0.06	0.06
		3	0.06	0.05	0.06	0.06	0.06	0.06
		4	0.06	0.06	0.05	0.06	0.06	0.06
		5	0.06	0.06	0.06	0.06	0.06	0.06
		10	0.05	0.05	0.06	0.06	0.06	0.06
	10	1.1	0.04	0.04	0.04	0.04	0.05	0.05
		1.5	0.05	0.05	0.04	0.04	0.04	0.04
		2	0.04	0.05	0.04	0.05	0.04	0.04
		3	0.05	0.05	0.04	0.04	0.04	0.04
		4	0.04	0.05	0.04	0.04	0.05	0.04
		5	0.04	0.04	0.04	0.05	0.04	0.04
		10	0.04	0.04	0.04	0.05	0.05	0.04

Notations: *n*: sample size; λ: mean of the null generating model (single Poisson).
Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 30 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.02 for each configuration.

Table A.2 – Simulated type I error rate of the LRT for testing ZIPs using the PPC method based 42 configurations of prior parameter values (4.1) (*N*= 500 replicates)

			Simulated Type I Error Rate (Kepner and Wackerly Values)					
			t					
n	λ	α	0.1	0.5	1	3	5	10
50	1	1.1	0.05	0.05	0.05	0.05	0.05	0.05
		1.5	0.05	0.05	0.05	0.05	0.05	0.06
		2	0.05	0.05	0.05	0.05	0.05	0.05
		3	0.05	0.05	0.05	0.05	0.06	0.05
		4	0.05	0.05	0.04	0.04	0.05	0.04
		5	0.05	0.05	0.04	0.04	0.04	0.05
		10	0.05	0.05	0.04	0.05	0.05	0.04
	2	1.1	0.05	0.05	0.05	0.05	0.05	0.05
		1.5	0.05	0.05	0.05	0.04	0.05	0.05
		2	0.05	0.05	0.05	0.05	0.05	0.04
		3	0.04	0.05	0.05	0.05	0.04	0.06
		4	0.05	0.05	0.05	0.05	0.05	0.05
		5	0.05	0.05	0.05	0.05	0.05	0.04
		10	0.05	0.05	0.05	0.04	0.06	0.05
	3	1.1	0.07	0.07	0.06	0.06	0.06	0.07
		1.5	0.06	0.06	0.07	0.07	0.07	0.07
		2	0.07	0.07	0.07	0.07	0.07	0.07
		3	0.07	0.07	0.06	0.06	0.06	0.06
		4	0.07	0.07	0.06	0.07	0.06	0.06
		5	0.07	0.07	0.07	0.06	0.07	0.07
		10	0.06	0.06	0.07	0.07	0.06	0.07

Notations: *n*: sample size; λ: mean of the null generating model (single Poisson).
Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 30 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.02 for each configuration.

Table A.2 (Continued) – Simulated type I error rate of the LRT for testing ZIPs using the PPC method based 42 configurations of prior parameter values (4.1) (*N*= 500 replicates)

			Simulated Type I Error Rate (Kepner and Wackerly Values)					
			t					
n	λ	α	0.1	0.5	1	3	5	10
100	1	1.1	0.05	0.05	0.05	0.05	0.05	0.05
		1.5	0.05	0.05	0.05	0.05	0.05	0.05
		2	0.05	0.05	0.05	0.05	0.05	0.05
		3	0.05	0.05	0.05	0.05	0.05	0.05
		4	0.05	0.06	0.05	0.05	0.06	0.05
		5	0.05	0.05	0.05	0.06	0.05	0.05
		10	0.05	0.06	0.06	0.05	0.05	0.06
	2	1.1	0.06	0.05	0.05	0.05	0.06	0.06
		1.5	0.06	0.06	0.05	0.06	0.06	0.06
		2	0.06	0.06	0.05	0.05	0.06	0.06
		3	0.06	0.05	0.05	0.06	0.06	0.07
		4	0.06	0.05	0.06	0.06	0.06	0.06
		5	0.06	0.06	0.06	0.06	0.06	0.06
		10	0.05	0.05	0.06	0.06	0.06	0.06
	3	1.1	0.06	0.05	0.06	0.07	0.06	0.05
		1.5	0.06	0.05	0.06	0.06	0.06	0.06
		2	0.05	0.06	0.06	0.06	0.06	0.05
		3	0.06	0.06	0.06	0.05	0.06	0.05
		4	0.05	0.06	0.06	0.05	0.06	0.05
		5	0.06	0.06	0.06	0.06	0.05	0.05
		10	0.06	0.05	0.05	0.06	0.06	0.06

Notations: *n*: sample size; λ: mean of the null generating model (single Poisson).
Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 30 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.02 for each configuration.

Table A.2 (Continued) – Simulated type I error rate of the LRT for testing ZIPs using the PPC method based 42 configurations of prior parameter values (4.1) (*N*= 500 replicates)

			Simulated Type I Error Rate (Kepner and Wackerly Values)					
			t					
n	λ	α	0.1	0.5	1	3	5	10
200	1	1.1	0.05	0.04	0.05	0.04	0.05	0.05
		1.5	0.04	0.04	0.05	0.04	0.05	0.04
		2	0.05	0.05	0.04	0.05	0.05	0.05
		3	0.05	0.04	0.05	0.05	0.05	0.04
		4	0.05	0.05	0.05	0.04	0.04	0.05
		5	0.05	0.04	0.04	0.05	0.05	0.04
		10	0.04	0.05	0.04	0.05	0.04	0.05
	2	1.1	0.06	0.06	0.06	0.06	0.06	0.06
		1.5	0.05	0.05	0.06	0.06	0.06	0.06
		2	0.06	0.06	0.06	0.06	0.06	0.06
		3	0.06	0.06	0.06	0.05	0.06	0.06
		4	0.06	0.06	0.05	0.05	0.06	0.06
		5	0.06	0.06	0.06	0.06	0.06	0.06
		10	0.06	0.06	0.05	0.06	0.05	0.06
	3	1.1	0.06	0.05	0.06	0.06	0.06	0.05
		1.5	0.06	0.06	0.06	0.06	0.05	0.05
		2	0.05	0.05	0.06	0.06	0.05	0.05
		3	0.06	0.05	0.05	0.05	0.05	0.05
		4	0.05	0.05	0.05	0.06	0.06	0.05
		5	0.05	0.05	0.05	0.05	0.06	0.06
		10	0.06	0.06	0.06	0.05	0.05	0.05

Notations: *n*: sample size; λ: mean of the null generating model (single Poisson).
Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 30 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.02 for each configuration.

Appendix B: Simulated power of the LRT using the PPC method

Table B.1 – Simulated power of the LRT for testing two-component Poisson mixtures using the PPC method based 42 configurations of prior parameter values (4.1) (*N*= 500 replicates)

						Simulated Power (Kepner and Wackerly Values) *T*					
π_1	*N*	λ_1	λ_2	*D*	α	0.1	0.5	1	3	5	10
0.1	50	1	4	3	1.1	0.35	0.36	0.36	0.37	0.36	0.36
					1.5	0.36	0.34	0.36	0.35	0.35	0.35
					2	0.37	0.36	0.35	0.37	0.35	0.35
					3	0.37	0.35	0.37	0.36	0.37	0.35
					4	0.36	0.35	0.36	0.35	0.35	0.36
					5	0.36	0.36	0.35	0.35	0.36	0.36
					10	0.36	0.35	0.35	0.36	0.36	0.37
		3	6	3	1.1	0.19	0.19	0.19	0.19	0.19	0.19
					1.5	0.19	0.19	0.19	0.18	0.19	0.19
					2	0.19	0.19	0.19	0.18	0.19	0.18
					3	0.20	0.19	0.18	0.19	0.19	0.18
					4	0.18	0.19	0.19	0.20	0.19	0.18
					5	0.18	0.19	0.19	0.19	0.19	0.18
					10	0.18	0.17	0.18	0.19	0.19	0.18
		2	4	2	1.1	0.13	0.14	0.14	0.13	0.14	0.14
					1.5	0.13	0.14	0.14	0.13	0.14	0.13
					2	0.14	0.13	0.13	0.13	0.14	0.13
					3	0.13	0.13	0.13	0.13	0.13	0.14
					4	0.14	0.14	0.13	0.14	0.13	0.13
					5	0.13	0.13	0.13	0.13	0.14	0.13
					10	0.14	0.13	0.14	0.14	0.13	0.13
		3	5	2	1.1	0.11	0.11	0.11	0.11	0.11	0.10
					1.5	0.10	0.12	0.11	0.11	0.11	0.11
					2	0.12	0.12	0.11	0.11	0.11	0.11
					3	0.11	0.11	0.11	0.12	0.11	0.11
					4	0.11	0.11	0.12	0.11	0.11	0.11
					5	0.11	0.11	0.11	0.12	0.11	0.11
					10	0.11	0.11	0.11	0.10	0.10	0.11

Notations: π_1: mixing proportion of the first component; *n*: sample size; λ_1: the first component mean; λ_2: the second component mean; *D*: $\lambda_2 - \lambda_1$.

Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 10 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.04 for each configuration.

Table B.1 (Continued) – Simulated power of the LRT for testing two-component Poisson mixtures using the PPC method based 42 configurations of prior parameter values (4.1) (*N*= 500 replicates)

						Simulated Power (Kepner and Wackerly Values) *t*					
π_1	*n*	λ_1	λ_2	*D*	α	0.1	0.5	1	3	5	10
0.1	100	1	4	3	1.1	0.52	0.53	0.52	0.52	0.52	0.52
					1.5	0.52	0.52	0.52	0.53	0.51	0.53
					2	0.52	0.52	0.52	0.52	0.52	0.52
					3	0.52	0.53	0.51	0.52	0.52	0.53
					4	0.51	0.52	0.51	0.52	0.52	0.52
					5	0.52	0.52	0.52	0.51	0.52	0.51
					10	0.53	0.53	0.52	0.50	0.51	0.53
		3	6	3	1.1	0.28	0.27	0.27	0.27	0.28	0.26
					1.5	0.28	0.28	0.27	0.27	0.27	0.27
					2	0.28	0.27	0.27	0.27	0.27	0.26
					3	0.28	0.27	0.27	0.27	0.27	0.26
					4	0.27	0.27	0.26	0.26	0.27	0.27
					5	0.26	0.27	0.26	0.27	0.26	0.27
					10	0.28	0.27	0.26	0.27	0.26	0.27
		2	4	2	1.1	0.18	0.19	0.19	0.19	0.18	0.18
					1.5	0.19	0.19	0.19	0.18	0.18	0.19
					2	0.18	0.18	0.19	0.18	0.19	0.18
					3	0.19	0.19	0.18	0.19	0.19	0.19
					4	0.19	0.18	0.18	0.19	0.19	0.18
					5	0.18	0.18	0.19	0.18	0.19	0.19
					10	0.19	0.18	0.18	0.18	0.18	0.18
		3	5	2	1.1	0.12	0.12	0.12	0.12	0.12	0.12
					1.5	0.12	0.12	0.12	0.12	0.12	0.13
					2	0.12	0.12	0.13	0.12	0.12	0.12
					3	0.12	0.12	0.12	0.12	0.12	0.13
					4	0.12	0.13	0.12	0.12	0.12	0.12
					5	0.12	0.12	0.13	0.12	0.12	0.12
					10	0.13	0.11	0.12	0.12	0.12	0.12

Notations: π_1: mixing proportion of the first component; *n*: sample size; λ_1: the first component mean; λ_2: the second component mean; *D*: $\lambda_2 - \lambda_1$.
Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 10 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.04 for each configuration.

Table B.1 (Continued) – Simulated power of the LRT for testing two-component Poisson mixtures using the PPC method based 42 configurations of prior parameter values (4.1) (*N*= 500 replicates)

						Simulated Power (Kepner and Wackerly Values)					
						t					
π_1	*n*	λ_1	λ_2	*D*	α	0.1	0.5	1	3	5	10
0.1	200	1	4	3	1.1	0.80	0.81	0.80	0.80	0.81	0.80
					1.5	0.80	0.80	0.80	0.80	0.80	0.80
					2	0.80	0.80	0.80	0.80	0.80	0.80
					3	0.81	0.81	0.80	0.80	0.80	0.79
					4	0.81	0.80	0.80	0.80	0.80	0.80
					5	0.80	0.80	0.80	0.80	0.80	0.80
					10	0.80	0.81	0.80	0.80	0.80	0.81
		3	6	3	1.1	0.40	0.41	0.40	0.41	0.41	0.40
					1.5	0.41	0.40	0.39	0.41	0.41	0.40
					2	0.39	0.41	0.41	0.41	0.41	0.41
					3	0.41	0.41	0.41	0.40	0.40	0.40
					4	0.40	0.41	0.40	0.41	0.40	0.40
					5	0.40	0.41	0.41	0.40	0.41	0.41
					10	0.41	0.40	0.4	0.41	0.41	0.40
		2	4	2	1.1	0.22	0.22	0.22	0.23	0.22	0.22
					1.5	0.21	0.23	0.22	0.22	0.22	0.22
					2	0.22	0.22	0.22	0.22	0.21	0.22
					3	0.22	0.22	0.21	0.22	0.23	0.22
					4	0.22	0.22	0.21	0.22	0.22	0.22
					5	0.22	0.22	0.22	0.20	0.22	0.22
					10	0.22	0.22	0.22	0.23	0.23	0.21
		3	5	2	1.1	0.18	0.18	0.17	0.18	0.17	0.17
					1.5	0.18	0.18	0.17	0.17	0.17	0.18
					2	0.18	0.18	0.18	0.18	0.17	0.17
					3	0.17	0.17	0.18	0.18	0.17	0.18
					4	0.18	0.18	0.18	0.17	0.17	0.17
					5	0.17	0.17	0.17	0.18	0.18	0.17
					10	0.17	0.18	0.17	0.18	0.18	0.17

Notations: π_1: mixing proportion of the first component; *n*: sample size; λ_1: the first component mean; λ_2: the second component mean; *D*: $\lambda_2 - \lambda_1$.
Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 10 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.04 for each configuration.

Table B.1 (Continued) – Simulated power of the LRT for testing two-component Poisson mixtures using the PPC method based 42 configurations of prior parameter values (4.1) (*N*= 500 replicates)

						Simulated Power (Kepner and Wackerly Values)					
						t					
π_1	*n*	λ_1	λ_2	*D*	α	0.1	0.5	1	3	5	10
0.3	50	1	4	3	1.1	0.82	0.83	0.83	0.82	0.83	0.83
					1.5	0.83	0.82	0.83	0.83	0.83	0.82
					2	0.82	0.82	0.83	0.82	0.83	0.82
					3	0.83	0.83	0.82	0.82	0.82	0.83
					4	0.83	0.83	0.83	0.82	0.83	0.83
					5	0.83	0.83	0.83	0.83	0.83	0.83
					10	0.82	0.83	0.83	0.83	0.82	0.81
		3	6	3	1.1	0.47	0.46	0.47	0.46	0.47	0.46
					1.5	0.47	0.47	0.48	0.48	0.47	0.48
					2	0.48	0.48	0.47	0.46	0.48	0.47
					3	0.47	0.46	0.47	0.47	0.46	0.48
					4	0.47	0.46	0.47	0.47	0.47	0.46
					5	0.47	0.48	0.47	0.47	0.48	0.47
					10	0.47	0.47	0.47	0.47	0.46	0.47
		2	4	2	1.1	0.32	0.32	0.33	0.33	0.33	0.33
					1.5	0.33	0.33	0.33	0.32	0.33	0.33
					2	0.33	0.33	0.33	0.33	0.32	0.32
					3	0.33	0.33	0.33	0.33	0.34	0.32
					4	0.33	0.33	0.34	0.33	0.33	0.34
					5	0.33	0.33	0.33	0.32	0.33	0.32
					10	0.34	0.32	0.33	0.33	0.33	0.32
		3	5	2	1.1	0.24	0.23	0.23	0.23	0.24	0.24
					1.5	0.24	0.24	0.23	0.23	0.23	0.23
					2	0.24	0.23	0.24	0.24	0.23	0.23
					3	0.24	0.22	0.24	0.23	0.23	0.23
					4	0.23	0.23	0.24	0.23	0.22	0.23
					5	0.22	0.23	0.23	0.23	0.23	0.23
					10	0.24	0.23	0.23	0.22	0.23	0.23

Notations: π_1: mixing proportion of the first component; *n*: sample size; λ_1: the first component mean; λ_2: the second component mean; *D*: $\lambda_2 - \lambda_1$.
Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 10 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.04 for each configuration.

Table B.1 (Continued) – Simulated power of the LRT for testing two-component Poisson mixtures using the PPC method based 42 configurations of prior parameter values (4.1) (*N*= 500 replicates)

						Simulated Power (Kepner and Wackerly Values)					
						t					
π_1	*n*	λ_1	λ_2	*D*	α	0.1	0.5	1	3	5	10
0.3	100	1	4	3	1.1	0.98	0.98	0.98	0.98	0.98	0.98
					1.5	0.98	0.98	0.98	0.98	0.98	0.99
					2	0.98	0.98	0.98	0.98	0.99	0.98
					3	0.98	0.99	0.98	0.98	0.99	0.98
					4	0.98	0.98	0.98	0.98	0.98	0.98
					5	0.98	0.98	0.98	0.98	0.98	0.99
					10	0.98	0.99	0.98	0.98	0.99	0.98
		3	6	3	1.1	0.70	0.70	0.70	0.71	0.71	0.71
					1.5	0.72	0.71	0.70	0.70	0.71	0.70
					2	0.71	0.71	0.70	0.71	0.70	0.70
					3	0.71	0.71	0.71	0.71	0.71	0.70
					4	0.72	0.71	0.70	0.71	0.71	0.71
					5	0.71	0.71	0.71	0.71	0.70	0.70
					10	0.70	0.70	0.71	0.71	0.72	0.70
		2	4	2	1.1	0.46	0.45	0.46	0.45	0.46	0.46
					1.5	0.45	0.45	0.46	0.46	0.47	0.46
					2	0.45	0.46	0.45	0.47	0.46	0.46
					3	0.46	0.45	0.45	0.47	0.46	0.46
					4	0.45	0.47	0.46	0.46	0.44	0.45
					5	0.46	0.45	0.45	0.45	0.46	0.45
					10	0.46	0.47	0.45	0.45	0.46	0.46
		3	5	2	1.1	0.30	0.29	0.29	0.29	0.30	0.29
					1.5	0.29	0.29	0.28	0.30	0.30	0.29
					2	0.30	0.30	0.29	0.29	0.30	0.29
					3	0.29	0.29	0.30	0.30	0.30	0.29
					4	0.30	0.29	0.30	0.29	0.29	0.30
					5	0.30	0.29	0.29	0.29	0.30	0.30
					10	0.30	0.29	0.30	0.30	0.29	0.30

Notations: π_1: mixing proportion of the first component; *n*: sample size; λ_1: the first component mean; λ_2: the second component mean; *D*: $\lambda_2 - \lambda_1$.

Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 10 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.04 for each configuration.

Table B.1 (Continued) – Simulated power of the LRT for testing two-component Poisson mixtures using the PPC method based 42 configurations of prior parameter values (4.1) (*N*= 500 replicates)

						Simulated Power (Kepner and Wackerly Values)					
						t					
π_1	*n*	λ_1	λ_2	*D*	α	0.1	0.5	1	3	5	10
0.3	200	1	4	3	1.1	1.00	1.00	1.00	1.00	1.00	1.00
					1.5	1.00	1.00	1.00	1.00	1.00	1.00
					2	1.00	1.00	1.00	1.00	1.00	1.00
					3	1.00	1.00	1.00	1.00	1.00	1.00
					4	1.00	1.00	1.00	1.00	1.00	1.00
					5	1.00	1.00	1.00	1.00	1.00	1.00
					10	1.00	1.00	1.00	1.00	1.00	1.00
		3	6	3	1.1	0.92	0.92	0.92	0.92	0.92	0.92
					1.5	0.92	0.92	0.92	0.92	0.92	0.92
					2	0.92	0.92	0.92	0.92	0.92	0.93
					3	0.92	0.92	0.93	0.93	0.93	0.93
					4	0.92	0.92	0.92	0.92	0.92	0.92
					5	0.92	0.92	0.92	0.92	0.92	0.92
					10	0.93	0.92	0.92	0.93	0.92	0.92
		2	4	2	1.1	0.72	0.72	0.73	0.72	0.72	0.71
					1.5	0.71	0.72	0.72	0.72	0.72	0.72
					2	0.72	0.73	0.72	0.71	0.73	0.72
					3	0.72	0.71	0.72	0.72	0.73	0.72
					4	0.72	0.72	0.73	0.73	0.71	0.73
					5	0.73	0.72	0.72	0.71	0.72	0.73
					10	0.72	0.72	0.73	0.71	0.73	0.72
		3	5	2	1.1	0.48	0.47	0.48	0.48	0.47	0.46
					1.5	0.48	0.47	0.48	0.48	0.47	0.47
					2	0.47	0.46	0.46	0.47	0.47	0.47
					3	0.47	0.47	0.47	0.47	0.47	0.48
					4	0.48	0.47	0.47	0.48	0.48	0.47
					5	0.48	0.48	0.49	0.48	0.47	0.46
					10	0.47	0.48	0.47	0.47	0.46	0.48

Notations: π_1: mixing proportion of the first component; *n*: sample size; λ_1: the first component mean; λ_2: the second component mean; *D*: $\lambda_2 - \lambda_1$.

Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 10 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.04 for each configuration.

Table B.1 (Continued) – Simulated power of the LRT for testing two-component Poisson mixtures using the PPC method based 42 configurations of prior parameter values (4.1) (*N*= 500 replicates)

						Simulated Power (Kepner and Wackerly Values)					
						t					
π_1	*n*	λ_1	λ_2	*D*	α	0.1	0.5	1	3	5	10
0.5	50	1	4	3	1.1	0.96	0.95	0.95	0.96	0.96	0.96
					1.5	0.96	0.95	0.96	0.96	0.96	0.95
					2	0.96	0.95	0.95	0.96	0.96	0.96
					3	0.96	0.96	0.95	0.96	0.96	0.96
					4	0.96	0.96	0.95	0.96	0.96	0.96
					5	0.96	0.96	0.96	0.96	0.96	0.95
					10	0.96	0.95	0.96	0.96	0.96	0.96
		3	6	3	1.1	0.63	0.64	0.63	0.63	0.63	0.63
					1.5	0.63	0.64	0.63	0.63	0.63	0.63
					2	0.64	0.64	0.62	0.63	0.63	0.63
					3	0.64	0.63	0.63	0.62	0.63	0.63
					4	0.63	0.63	0.64	0.63	0.62	0.64
					5	0.64	0.63	0.63	0.63	0.63	0.63
					10	0.64	0.64	0.64	0.63	0.62	0.63
		2	4	2	1.1	0.40	0.40	0.41	0.41	0.40	0.40
					1.5	0.41	0.39	0.41	0.40	0.40	0.40
					2	0.41	0.41	0.40	0.41	0.40	0.40
					3	0.40	0.40	0.39	0.40	0.40	0.40
					4	0.40	0.40	0.40	0.40	0.41	0.41
					5	0.41	0.40	0.41	0.41	0.41	0.41
					10	0.40	0.41	0.41	0.40	0.40	0.39
		3	5	2	1.1	0.28	0.27	0.28	0.28	0.28	0.28
					1.5	0.27	0.27	0.27	0.27	0.28	0.27
					2	0.28	0.28	0.27	0.28	0.28	0.27
					3	0.28	0.28	0.27	0.28	0.28	0.28
					4	0.29	0.28	0.27	0.27	0.27	0.28
					5	0.28	0.28	0.28	0.28	0.28	0.28
					10	0.28	0.29	0.28	0.29	0.29	0.28

Notations: π_1: mixing proportion of the first component; *n*: sample size; λ_1: the first component mean; λ_2: the second component mean; *D*: $\lambda_2 - \lambda_1$.

Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 10 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.04 for each configuration.

Table B.1 (Continued) – Simulated power of the LRT for testing two-component Poisson mixtures using the PPC method based 42 configurations of prior parameter values (4.1) (*N*= 500 replicates)

						Simulated Power (Kepner and Wackerly Values)					
						t					
π_1	*n*	λ_1	λ_2	*D*	α	0.1	0.5	1	3	5	10
					1.1	1.00	1.00	1.00	1.00	1.00	1.00
					1.5	1.00	1.00	1.00	1.00	1.00	1.00
					2	1.00	1.00	1.00	1.00	1.00	1.00
		1	4	3	3	1.00	1.00	1.00	1.00	1.00	1.00
					4	1.00	1.00	1.00	1.00	1.00	1.00
					5	1.00	1.00	1.00	1.00	1.00	1.00
					10	1.00	1.00	1.00	1.00	1.00	1.00
					1.1	0.84	0.84	0.84	0.84	0.84	0.84
					1.5	0.84	0.85	0.84	0.84	0.84	0.84
					2	0.85	0.84	0.83	0.84	0.84	0.84
		3	6	3	3	0.84	0.84	0.84	0.85	0.85	0.84
					4	0.85	0.85	0.84	0.84	0.84	0.84
					5	0.84	0.84	0.84	0.85	0.84	0.84
0.5	100				10	0.84	0.84	0.84	0.83	0.84	0.85
					1.1	0.60	0.59	0.60	0.59	0.59	0.59
					1.5	0.60	0.60	0.60	0.59	0.59	0.59
					2	0.59	0.61	0.59	0.60	0.59	0.59
		2	4	2	3	0.60	0.60	0.60	0.59	0.60	0.59
					4	0.59	0.60	0.60	0.59	0.60	0.59
					5	0.59	0.59	0.60	0.59	0.59	0.59
					10	0.60	0.59	0.60	0.59	0.60	0.59
					1.1	0.43	0.44	0.45	0.44	0.45	0.44
					1.5	0.44	0.44	0.45	0.42	0.44	0.44
					2	0.44	0.44	0.44	0.44	0.45	0.45
		3	5	2	3	0.45	0.44	0.45	0.43	0.44	0.45
					4	0.45	0.45	0.45	0.45	0.43	0.45
					5	0.45	0.45	0.43	0.43	0.43	0.43
					10	0.46	0.44	0.43	0.44	0.44	0.44

Notations: π_1: mixing proportion of the first component; *n*: sample size; λ_1: the first component mean; λ_2: the second component mean; *D*: $\lambda_2 - \lambda_1$.

Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 10 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.04 for each configuration.

Table B.1 (Continued) – Simulated power of the LRT for testing two-component Poisson mixtures using the PPC method based 42 configurations of prior parameter values (4.1) (N= 500 replicates)

						Simulated Power (Kepner and Wackerly Values)					
						t					
π_1	n	λ_1	λ_2	D	α	0.1	0.5	1	3	5	10
0.5	200	1	4	3	1.1	1.00	1.00	1.00	1.00	1.00	1.00
					1.5	1.00	1.00	1.00	1.00	1.00	1.00
					2	1.00	1.00	1.00	1.00	1.00	1.00
					3	1.00	1.00	1.00	1.00	1.00	1.00
					4	1.00	1.00	1.00	1.00	1.00	1.00
					5	1.00	1.00	1.00	1.00	1.00	1.00
					10	1.00	1.00	1.00	1.00	1.00	1.00
		3	6	3	1.1	0.98	0.99	0.98	0.99	0.98	0.98
					1.5	0.99	0.99	0.98	0.98	0.99	0.98
					2	0.99	0.99	0.99	0.98	0.98	0.98
					3	0.99	0.98	0.99	0.99	0.99	0.99
					4	0.98	0.98	0.98	0.99	0.99	0.99
					5	0.99	0.99	0.98	0.99	0.99	0.98
					10	0.99	0.98	0.98	0.98	0.98	0.99
		2	4	2	1.1	0.87	0.88	0.87	0.87	0.87	0.87
					1.5	0.86	0.88	0.87	0.87	0.87	0.88
					2	0.87	0.88	0.87	0.88	0.87	0.87
					3	0.87	0.88	0.87	0.87	0.87	0.87
					4	0.86	0.86	0.87	0.87	0.87	0.87
					5	0.87	0.87	0.87	0.87	0.86	0.87
					10	0.87	0.87	0.88	0.87	0.87	0.87
		3	5	2	1.1	0.67	0.67	0.67	0.67	0.67	0.68
					1.5	0.67	0.66	0.66	0.67	0.66	0.66
					2	0.66	0.66	0.65	0.66	0.66	0.67
					3	0.67	0.66	0.67	0.67	0.66	0.66
					4	0.67	0.67	0.66	0.68	0.67	0.66
					5	0.66	0.66	0.67	0.67	0.65	0.66
					10	0.67	0.67	0.68	0.66	0.66	0.66

Notations: π_1: mixing proportion of the first component; n: sample size; λ_1: the first component mean; λ_2: the second component mean; D: $\lambda_2 - \lambda_1$.

Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 10 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.04 for each configuration.

Table B.1 (Continued) – Simulated power of the LRT for testing two-component Poisson mixtures using the PPC method based 42 configurations of prior parameter values (4.1) (*N*= 500 replicates)

						Simulated Power (Kepner and Wackerly Values)					
						t					
π_1	*n*	λ_1	λ_2	*D*	α	0.1	0.5	1	3	5	10
0.7	50	1	4	3	1.1	0.95	0.95	0.95	0.96	0.95	0.96
					1.5	0.96	0.95	0.95	0.96	0.95	0.95
					2	0.95	0.96	0.96	0.96	0.96	0.95
					3	0.95	0.96	0.96	0.95	0.95	0.95
					4	0.96	0.95	0.96	0.96	0.96	0.95
					5	0.95	0.95	0.95	0.95	0.95	0.95
					10	0.95	0.95	0.95	0.95	0.95	0.95
		3	6	3	1.1	0.61	0.62	0.61	0.62	0.61	0.61
					1.5	0.61	0.62	0.62	0.62	0.61	0.6
					2	0.62	0.62	0.62	0.63	0.61	0.62
					3	0.62	0.61	0.62	0.60	0.62	0.60
					4	0.61	0.61	0.61	0.61	0.61	0.60
					5	0.62	0.61	0.61	0.60	0.61	0.61
					10	0.62	0.61	0.61	0.61	0.61	0.61
		2	4	2	1.1	0.38	0.37	0.37	0.37	0.37	0.37
					1.5	0.36	0.37	0.37	0.37	0.37	0.36
					2	0.37	0.36	0.37	0.36	0.37	0.36
					3	0.36	0.37	0.37	0.37	0.37	0.37
					4	0.37	0.37	0.37	0.37	0.36	0.37
					5	0.37	0.36	0.37	0.36	0.37	0.36
					10	0.37	0.37	0.35	0.35	0.36	0.36
		3	5	2	1.1	0.26	0.26	0.26	0.25	0.26	0.25
					1.5	0.26	0.26	0.25	0.26	0.26	0.25
					2	0.24	0.26	0.26	0.26	0.26	0.26
					3	0.25	0.25	0.25	0.25	0.26	0.27
					4	0.26	0.25	0.26	0.26	0.25	0.26
					5	0.26	0.27	0.26	0.27	0.25	0.25
					10	0.26	0.26	0.25	0.25	0.24	0.25

Notations: π_1: mixing proportion of the first component; *n*: sample size; λ_1: the first component mean; λ_2: the second component mean; *D*: $\lambda_2 - \lambda_1$.

Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 10 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.04 for each configuration.

Table B.1 (Continued) – Simulated power of the LRT for testing two-component Poisson mixtures using the PPC method based 42 configurations of prior parameter values (4.1) (*N*= 500 replicates)

						Simulated Power (Kepner and Wackerly Values)					
						t					
π_1	*n*	λ_1	λ_2	*D*	α	0.1	0.5	1	3	5	10
0.7	100	1	4	3	1.1	1.00	1.00	1.00	1.00	1.00	1.00
					1.5	1.00	1.00	1.00	1.00	1.00	1.00
					2	1.00	1.00	1.00	1.00	1.00	1.00
					3	1.00	1.00	1.00	1.00	1.00	1.00
					4	1.00	1.00	1.00	1.00	1.00	1.00
					5	1.00	1.00	1.00	1.00	1.00	1.00
					10	1.00	1.00	1.00	1.00	1.00	1.00
		3	6	3	1.1	0.79	0.79	0.79	0.80	0.79	0.79
					1.5	0.79	0.79	0.80	0.80	0.79	0.78
					2	0.79	0.79	0.78	0.80	0.78	0.79
					3	0.80	0.78	0.79	0.79	0.80	0.79
					4	0.80	0.78	0.79	0.79	0.79	0.79
					5	0.80	0.79	0.79	0.79	0.78	0.80
					10	0.79	0.79	0.79	0.79	0.78	0.79
		2	4	2	1.1	0.57	0.56	0.56	0.57	0.57	0.58
					1.5	0.57	0.56	0.58	0.57	0.57	0.56
					2	0.57	0.57	0.57	0.57	0.56	0.57
					3	0.56	0.57	0.57	0.57	0.56	0.56
					4	0.56	0.56	0.56	0.56	0.57	0.56
					5	0.57	0.56	0.57	0.57	0.57	0.57
					10	0.56	0.56	0.56	0.56	0.57	0.56
		3	5	2	1.1	0.41	0.41	0.40	0.40	0.39	0.40
					1.5	0.40	0.40	0.39	0.41	0.40	0.40
					2	0.40	0.40	0.39	0.40	0.41	0.41
					3	0.41	0.40	0.40	0.41	0.39	0.40
					4	0.40	0.41	0.40	0.40	0.40	0.41
					5	0.41	0.39	0.40	0.40	0.40	0.40
					10	0.41	0.40	0.40	0.40	0.40	0.41

Notations: π_1: mixing proportion of the first component; *n*: sample size; λ_1: the first component mean; λ_2: the second component mean; *D*: $\lambda_2 - \lambda_1$.

Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 10 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.04 for each configuration.

Table B.1 (Continued) – Simulated power of the LRT for testing two-component Poisson mixtures using the PPC method based 42 configurations of prior parameter values (4.1) (*N*= 500 replicates)

						Simulated Power (Kepner and Wackerly Values)					
						t					
π_1	*n*	λ_1	λ_2	*D*	α	0.1	0.5	1	3	5	10
0.7	200	1	4	3	1.1	1.00	1.00	1.00	1.00	1.00	1.00
					1.5	1.00	1.00	1.00	1.00	1.00	1.00
					2	1.00	1.00	1.00	1.00	1.00	1.00
					3	1.00	1.00	1.00	1.00	1.00	1.00
					4	1.00	1.00	1.00	1.00	1.00	1.00
					5	1.00	1.00	1.00	1.00	1.00	1.00
					10	1.00	1.00	1.00	1.00	1.00	1.00
		3	6	3	1.1	0.98	0.98	0.98	0.98	0.98	0.98
					1.5	0.98	0.98	0.98	0.98	0.98	0.98
					2	0.98	0.98	0.98	0.98	0.98	0.98
					3	0.98	0.98	0.98	0.98	0.98	0.98
					4	0.98	0.98	0.98	0.98	0.98	0.98
					5	0.98	0.98	0.98	0.98	0.98	0.98
					10	0.98	0.98	0.98	0.98	0.98	0.98
		2	4	2	1.1	0.81	0.81	0.81	0.81	0.81	0.81
					1.5	0.81	0.81	0.81	0.81	0.81	0.80
					2	0.81	0.81	0.82	0.81	0.82	0.81
					3	0.81	0.81	0.81	0.81	0.80	0.80
					4	0.81	0.81	0.81	0.81	0.81	0.81
					5	0.80	0.81	0.81	0.80	0.81	0.81
					10	0.82	0.82	0.81	0.81	0.82	0.81
		3	5	2	1.1	0.63	0.63	0.62	0.63	0.62	0.63
					1.5	0.62	0.63	0.62	0.63	0.63	0.63
					2	0.62	0.63	0.62	0.63	0.63	0.63
					3	0.63	0.63	0.63	0.63	0.62	0.63
					4	0.62	0.62	0.64	0.62	0.62	0.63
					5	0.64	0.62	0.63	0.63	0.62	0.63
					10	0.62	0.62	0.62	0.63	0.64	0.63

Notations: π_1: mixing proportion of the first component; *n*: sample size; λ_1: the first component mean; λ_2: the second component mean; *D*: $\lambda_2 - \lambda_1$.
Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 10 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.04 for each configuration.

Table B.1 (Continued) – Simulated power of the LRT for testing two-component Poisson mixtures using the PPC method based 42 configurations of prior parameter values (4.1) (*N*= 500 replicates)

						Simulated Power (Kepner and Wackerly Values)					
						t					
π_1	*n*	λ_1	λ_2	*D*	α	0.1	0.5	1	3	5	10
0.9	50	1	4	3	1.1	0.62	0.62	0.63	0.62	0.62	0.62
					1.5	0.62	0.62	0.62	0.63	0.62	0.63
					2	0.62	0.63	0.63	0.61	0.61	0.62
					3	0.61	0.62	0.62	0.61	0.62	0.61
					4	0.63	0.61	0.62	0.62	0.61	0.61
					5	0.62	0.61	0.62	0.61	0.62	0.62
					10	0.62	0.62	0.62	0.61	0.62	0.61
		3	6	3	1.1	0.28	0.29	0.29	0.29	0.29	0.29
					1.5	0.28	0.28	0.29	0.29	0.28	0.28
					2	0.28	0.30	0.28	0.28	0.29	0.28
					3	0.28	0.29	0.28	0.29	0.28	0.29
					4	0.28	0.29	0.28	0.28	0.28	0.28
					5	0.28	0.28	0.29	0.28	0.29	0.29
					10	0.29	0.29	0.29	0.28	0.28	0.27
		2	4	2	1.1	0.18	0.18	0.18	0.18	0.18	0.18
					1.5	0.18	0.18	0.18	0.18	0.18	0.17
					2	0.17	0.18	0.18	0.18	0.18	0.19
					3	0.17	0.18	0.19	0.18	0.19	0.18
					4	0.17	0.17	0.18	0.19	0.18	0.17
					5	0.17	0.17	0.17	0.18	0.18	0.18
					10	0.19	0.18	0.18	0.18	0.18	0.18
		3	5	2	1.1	0.12	0.13	0.13	0.13	0.13	0.13
					1.5	0.12	0.13	0.13	0.13	0.12	0.13
					2	0.13	0.13	0.13	0.13	0.13	0.13
					3	0.14	0.13	0.13	0.12	0.13	0.13
					4	0.12	0.12	0.12	0.14	0.13	0.13
					5	0.13	0.13	0.13	0.12	0.12	0.13
					10	0.13	0.12	0.12	0.12	0.12	0.12

Notations: π_1: mixing proportion of the first component; *n*: sample size; λ_1: the first component mean; λ_2: the second component mean; *D*: $\lambda_2 - \lambda_1$.
Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 10 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.04 for each configuration.

Table B.1 (Continued) – Simulated power of the LRT for testing two-component Poisson mixtures using the PPC method based 42 configurations of prior parameter values (4.1) (*N*= 500 replicates)

						Simulated Power (Kepner and Wackerly Values)					
						t					
π_1	*n*	λ_1	λ_2	*D*	α	0.1	0.5	1	3	5	10
					1.1	0.89	0.88	0.88	0.88	0.88	0.88
					1.5	0.88	0.88	0.88	0.9	0.88	0.88
					2	0.88	0.89	0.89	0.89	0.89	0.89
		1	4	3	3	0.89	0.89	0.89	0.89	0.89	0.88
					4	0.89	0.88	0.88	0.90	0.88	0.88
					5	0.88	0.89	0.89	0.89	0.88	0.89
					10	0.88	0.88	0.89	0.88	0.88	0.89
					1.1	0.38	0.38	0.38	0.38	0.37	0.38
					1.5	0.37	0.38	0.38	0.37	0.38	0.38
					2	0.37	0.38	0.38	0.37	0.37	0.38
		3	6	3	3	0.37	0.38	0.37	0.38	0.37	0.38
					4	0.38	0.37	0.37	0.38	0.38	0.38
					5	0.38	0.38	0.38	0.38	0.38	0.37
0.9	100				10	0.37	0.38	0.38	0.37	0.37	0.37
					1.1	0.29	0.29	0.29	0.29	0.29	0.29
					1.5	0.29	0.29	0.29	0.29	0.28	0.29
					2	0.29	0.29	0.29	0.28	0.29	0.29
		2	4	2	3	0.28	0.29	0.29	0.29	0.29	0.29
					4	0.29	0.29	0.28	0.28	0.28	0.28
					5	0.28	0.28	0.28	0.28	0.28	0.28
					10	0.29	0.28	0.29	0.28	0.29	0.28
					1.1	0.19	0.19	0.19	0.18	0.19	0.19
					1.5	0.19	0.18	0.18	0.18	0.19	0.19
					2	0.19	0.18	0.18	0.19	0.18	0.19
		3	5	2	3	0.19	0.18	0.18	0.17	0.18	0.19
					4	0.19	0.18	0.19	0.19	0.19	0.18
					5	0.19	0.18	0.19	0.20	0.17	0.18
					10	0.19	0.18	0.19	0.17	0.19	0.19

Notations: π_1: mixing proportion of the first component; *n*: sample size; λ_1: the first component mean; λ_2: the second component mean; *D*: $\lambda_2 - \lambda_1$.
Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 10 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.04 for each configuration.

Table B.1 (Continued) – Simulated power of the LRT for testing two-component Poisson mixtures using the PPC method based 42 configurations of prior parameter values (4.1) (*N*= 500 replicates)

						Simulated Power (Kepner and Wackerly Values)					
						t					
π_1	*n*	λ_1	λ_2	*D*	α	0.1	0.5	1	3	5	10
0.9	200	1	4	3	1.1	0.99	0.99	0.99	0.99	0.99	0.99
					1.5	0.99	0.99	0.99	0.99	0.99	0.99
					2	0.99	0.99	0.99	0.99	0.99	0.99
					3	0.99	0.99	0.99	0.99	0.99	0.99
					4	0.99	0.99	0.99	0.99	0.99	0.99
					5	0.99	0.99	0.99	0.99	0.99	0.99
					10	0.99	0.99	0.99	0.99	0.99	0.99
		3	6	3	1.1	0.68	0.69	0.69	0.70	0.69	0.69
					1.5	0.70	0.70	0.69	0.69	0.69	0.69
					2	0.70	0.70	0.68	0.70	0.69	0.70
					3	0.69	0.69	0.68	0.69	0.69	0.70
					4	0.69	0.69	0.69	0.69	0.69	0.70
					5	0.69	0.68	0.70	0.69	0.69	0.69
					10	0.69	0.69	0.69	0.69	0.70	0.70
		2	4	2	1.1	0.43	0.42	0.43	0.43	0.43	0.43
					1.5	0.43	0.43	0.43	0.43	0.43	0.43
					2	0.42	0.43	0.42	0.44	0.43	0.42
					3	0.43	0.43	0.43	0.43	0.43	0.43
					4	0.44	0.44	0.42	0.42	0.43	0.43
					5	0.43	0.43	0.44	0.42	0.43	0.43
					10	0.42	0.43	0.43	0.42	0.43	0.42
		3	5	2	1.1	0.30	0.30	0.30	0.30	0.30	0.30
					1.5	0.31	0.31	0.29	0.29	0.30	0.31
					2	0.30	0.30	0.29	0.30	0.30	0.31
					3	0.31	0.31	0.30	0.31	0.31	0.30
					4	0.30	0.30	0.30	0.30	0.29	0.30
					5	0.30	0.30	0.30	0.30	0.30	0.31
					10	0.31	0.31	0.29	0.30	0.30	0.30

Notations: π_1: mixing proportion of the first component; *n*: sample size; λ_1: the first component mean; λ_2: the second component mean; *D*: $\lambda_2 - \lambda_1$.

Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 10 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.04 for each configuration.

Table B.2 **–** Simulated power of the LRT for testing ZIPs using the PPC method based 42 configurations of prior parameter values (4.1) (*N*= 500 replicates)

				Simulated Power (Kepner and Wackerly Values)					
				t					
π_1	*n*	λ_2	α	0.1	0.5	1	3	5	10
0.1	50	1	1.1	0.15	0.15	0.15	0.15	0.15	0.15
			1.5	0.14	0.14	0.14	0.15	0.14	0.14
			2	0.15	0.14	0.15	0.15	0.14	0.14
			3	0.14	0.15	0.14	0.14	0.14	0.14
			4	0.15	0.14	0.14	0.14	0.15	0.16
			5	0.15	0.15	0.14	0.14	0.15	0.14
			10	0.14	0.15	0.15	0.14	0.14	0.14
		2	1.1	0.39	0.40	0.40	0.39	0.38	0.40
			1.5	0.41	0.40	0.40	0.41	0.41	0.40
			2	0.40	0.40	0.40	0.40	0.40	0.39
			3	0.41	0.40	0.41	0.41	0.41	0.40
			4	0.40	0.40	0.40	0.40	0.39	0.39
			5	0.40	0.40	0.41	0.40	0.40	0.40
			10	0.41	0.41	0.40	0.41	0.41	0.41
		3	1.1	0.74	0.74	0.74	0.73	0.73	0.73
			1.5	0.73	0.73	0.74	0.74	0.74	0.74
			2	0.73	0.73	0.74	0.73	0.74	0.73
			3	0.73	0.74	0.74	0.73	0.74	0.73
			4	0.73	0.73	0.73	0.73	0.73	0.74
			5	0.74	0.74	0.74	0.73	0.74	0.73
			10	0.74	0.73	0.74	0.73	0.74	0.73

Notations: π_1: mixing proportion of the first component; *n*: sample size; λ_2: the second component mean.
Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 10 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.04 for each configuration.

Table B.2 (Continued) – Simulated power of the LRT for testing ZIPs using the PPC method based 42 configurations of prior parameter values (4.1) (N= 500 replicates)

				Simulated Power (Kepner and Wackerly Values)					
				t					
π_1	n	λ_2	α	0.1	0.5	1	3	5	10
0.1	100	1	1.1	0.22	0.23	0.23	0.23	0.23	0.22
			1.5	0.22	0.23	0.23	0.23	0.23	0.23
			2	0.22	0.22	0.23	0.23	0.23	0.24
			3	0.23	0.24	0.24	0.25	0.23	0.24
			4	0.23	0.24	0.22	0.24	0.23	0.24
			5	0.23	0.24	0.24	0.25	0.24	0.24
			10	0.24	0.22	0.23	0.24	0.23	0.23
		2	1.1	0.61	0.60	0.61	0.62	0.61	0.62
			1.5	0.61	0.60	0.61	0.61	0.60	0.60
			2	0.61	0.61	0.61	0.62	0.61	0.60
			3	0.60	0.61	0.60	0.60	0.60	0.60
			4	0.60	0.61	0.60	0.61	0.60	0.60
			5	0.60	0.59	0.61	0.60	0.62	0.60
			10	0.61	0.61	0.61	0.61	0.61	0.61
		3	1.1	0.92	0.92	0.92	0.92	0.92	0.92
			1.5	0.92	0.93	0.92	0.92	0.92	0.92
			2	0.91	0.92	0.92	0.92	0.92	0.92
			3	0.92	0.92	0.92	0.93	0.92	0.92
			4	0.92	0.92	0.93	0.92	0.92	0.92
			5	0.92	0.92	0.92	0.92	0.92	0.93
			10	0.92	0.92	0.92	0.92	0.92	0.92

Notations: π_1: mixing proportion of the first component; n: sample size; λ_2: the second component mean.
Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 10 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.04 for each configuration.

Table B.2 (Continued) – Simulated power of the LRT for testing ZIPs using the PPC method based 42 configurations of prior parameter values (4.1) (*N*= 500 replicates)

				Simulated Power (Kepner and Wackerly Values)					
				t					
π_1	*n*	λ_2	α	0.1	0.5	1	3	5	10
0.1	200	1	1.1	0.30	0.30	0.31	0.30	0.32	0.30
			1.5	0.31	0.30	0.30	0.30	0.30	0.30
			2	0.30	0.32	0.30	0.31	0.30	0.30
			3	0.31	0.31	0.31	0.30	0.30	0.31
			4	0.29	0.29	0.30	0.31	0.30	0.31
			5	0.31	0.30	0.31	0.31	0.30	0.30
			10	0.30	0.30	0.31	0.30	0.30	0.30
		2	1.1	0.84	0.83	0.82	0.83	0.82	0.83
			1.5	0.83	0.83	0.84	0.83	0.83	0.84
			2	0.83	0.83	0.83	0.83	0.83	0.84
			3	0.84	0.83	0.83	0.82	0.83	0.83
			4	0.84	0.83	0.83	0.83	0.83	0.83
			5	0.84	0.82	0.83	0.83	0.83	0.83
			10	0.83	0.83	0.83	0.83	0.83	0.83
		3	1.1	0.99	0.99	0.99	0.99	0.99	0.99
			1.5	0.99	0.99	0.99	0.99	0.99	0.99
			2	0.99	0.99	0.99	0.99	0.99	0.99
			3	0.99	0.99	0.99	0.99	0.99	0.99
			4	0.99	0.99	0.99	0.99	0.99	0.99
			5	0.99	0.99	0.99	0.99	0.99	0.99
			10	0.99	0.99	0.99	0.99	0.99	0.99

Notations: π_1: mixing proportion of the first component; *n*: sample size; λ_2: the second component mean.
Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 10 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.04 for each configuration.

Table B.2 (Continued) – Simulated power of the LRT for testing ZIPs using the PPC method based 42 configurations of prior parameter values (4.1) (N= 500 replicates)

				Simulated Power (Kepner and Wackerly Values)					
				t					
π_1	n	λ_2	α	0.1	0.5	1	3	5	10
			1.1	0.43	0.43	0.43	0.43	0.43	0.43
			1.5	0.43	0.43	0.42	0.43	0.44	0.43
			2	0.43	0.42	0.43	0.42	0.42	0.43
		1	3	0.43	0.42	0.43	0.44	0.42	0.42
			4	0.43	0.43	0.42	0.42	0.42	0.42
			5	0.42	0.42	0.42	0.42	0.42	0.42
			10	0.44	0.43	0.42	0.44	0.44	0.43
			1.1	0.94	0.94	0.94	0.94	0.94	0.94
			1.5	0.94	0.94	0.94	0.94	0.94	0.94
			2	0.94	0.94	0.94	0.93	0.94	0.93
0.3	50	2	3	0.94	0.94	0.94	0.94	0.94	0.93
			4	0.93	0.94	0.93	0.94	0.94	0.93
			5	0.94	0.94	0.93	0.93	0.94	0.93
			10	0.93	0.94	0.94	0.94	0.93	0.93
			1.1	1.00	1.00	1.00	1.00	1.00	1.00
			1.5	1.00	1.00	1.00	1.00	1.00	1.00
			2	1.00	1.00	1.00	1.00	1.00	1.00
		3	3	1.00	1.00	1.00	1.00	1.00	1.00
			4	1.00	1.00	1.00	1.00	1.00	1.00
			5	1.00	1.00	1.00	1.00	1.00	1.00
			10	1.00	1.00	1.00	1.00	1.00	1.00

Notations: π_1: mixing proportion of the first component; n: sample size; λ_2: the second component mean.
Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 10 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.04 for each configuration.

Table B.2 (Continued) – Simulated power of the LRT for testing ZIPs using the PPC method based 42 configurations of prior parameter values (4.1) (*N*= 500 replicates)

				Simulated Power (Kepner and Wackerly Values)					
				t					
π_1	*n*	λ_2	α	0.1	0.5	1	3	5	10
0.3	100	1	1.1	0.68	0.67	0.69	0.69	0.68	0.68
			1.5	0.68	0.68	0.67	0.68	0.67	0.67
			2	0.68	0.67	0.68	0.67	0.68	0.68
			3	0.68	0.67	0.69	0.67	0.68	0.68
			4	0.68	0.69	0.67	0.67	0.68	0.68
			5	0.69	0.68	0.68	0.68	0.68	0.68
			10	0.68	0.67	0.67	0.68	0.68	0.68
		2	1.1	0.99	0.99	1.00	0.99	1.00	1.00
			1.5	1.00	0.99	0.99	1.00	1.00	1.00
			2	1.00	1.00	1.00	1.00	0.99	1.00
			3	1.00	0.99	1.00	0.99	0.99	1.00
			4	0.99	1.00	1.00	1.00	0.99	1.00
			5	0.99	1.00	1.00	1.00	1.00	1.00
			10	1.00	1.00	1.00	1.00	0.99	1.00
		3	1.1	1.00	1.00	1.00	1.00	1.00	1.00
			1.5	1.00	1.00	1.00	1.00	1.00	1.00
			2	1.00	1.00	1.00	1.00	1.00	1.00
			3	1.00	1.00	1.00	1.00	1.00	1.00
			4	1.00	1.00	1.00	1.00	1.00	1.00
			5	1.00	1.00	1.00	1.00	1.00	1.00
			10	1.00	1.00	1.00	1.00	1.00	1.00

Notations: π_1: mixing proportion of the first component; *n*: sample size; λ_2: the second component mean.
Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 10 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.04 for each configuration.

Table B.2 (Continued) – Simulated power of the LRT for testing ZIPs using the PPC method based 42 configurations of prior parameter values (4.1) (*N*= 500 replicates)

				Simulated Power (Kepner and Wackerly Values)					
				t					
π_1	*n*	λ_2	α	0.1	0.5	1	3	5	10
0.3	200	1	1.1	0.93	0.92	0.93	0.93	0.93	0.93
			1.5	0.93	0.93	0.93	0.93	0.93	0.29
			2	0.93	0.93	0.93	0.93	0.93	0.93
			3	0.92	0.92	0.93	0.93	0.93	0.93
			4	0.92	0.92	0.93	0.93	0.93	0.92
			5	0.93	0.93	0.93	0.93	0.92	0.93
			10	0.92	0.93	0.94	0.93	0.93	0.93
		2	1.1	1.00	1.00	1.00	1.00	1.00	1.00
			1.5	1.00	1.00	1.00	1.00	1.00	1.00
			2	1.00	1.00	1.00	1.00	1.00	1.00
			3	1.00	1.00	1.00	1.00	1.00	1.00
			4	1.00	1.00	1.00	1.00	1.00	1.00
			5	1.00	1.00	1.00	1.00	1.00	1.00
			10	1.00	1.00	1.00	1.00	1.00	1.00
		3	1.1	1.00	1.00	1.00	1.00	1.00	1.00
			1.5	1.00	1.00	1.00	1.00	1.00	1.00
			2	1.00	1.00	1.00	1.00	1.00	1.00
			3	1.00	1.00	1.00	1.00	1.00	1.00
			4	1.00	1.00	1.00	1.00	1.00	1.00
			5	1.00	1.00	1.00	1.00	1.00	1.00
			10	1.00	1.00	1.00	1.00	1.00	1.00

Notations: π_1: mixing proportion of the first component; *n*: sample size; λ_2: the second component mean.
Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 10 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.04 for each configuration.

Table B.2 (Continued) – Simulated power of the LRT for testing ZIPs using the PPC method based 42 configurations of prior parameter values (4.1) (N= 500 replicates)

				Simulated Power (Kepner and Wackerly Values)					
				t					
π_1	n	λ_2	α	0.1	0.5	1	3	5	10
0.5	50	1	1.1	0.72	0.72	0.72	0.72	0.72	0.72
			1.5	0.72	0.71	0.71	0.72	0.71	0.71
			2	0.73	0.72	0.72	0.71	0.71	0.72
			3	0.71	0.71	0.70	0.71	0.71	0.71
			4	0.72	0.71	0.73	0.71	0.72	0.72
			5	0.72	0.72	0.71	0.71	0.70	0.72
			10	0.70	0.71	0.71	0.72	0.72	0.71
		2	1.1	1.00	1.00	1.00	1.00	1.00	1.00
			1.5	1.00	1.00	1.00	1.00	1.00	1.00
			2	1.00	1.00	1.00	1.00	1.00	1.00
			3	1.00	1.00	1.00	1.00	1.00	1.00
			4	1.00	1.00	1.00	1.00	1.00	1.00
			5	1.00	1.00	1.00	1.00	1.00	1.00
			10	1.00	1.00	1.00	1.00	1.00	1.00
		3	1.1	1.00	1.00	1.00	1.00	1.00	1.00
			1.5	1.00	1.00	1.00	1.00	1.00	1.00
			2	1.00	1.00	1.00	1.00	1.00	1.00
			3	1.00	1.00	1.00	1.00	1.00	1.00
			4	1.00	1.00	1.00	1.00	1.00	1.00
			5	1.00	1.00	1.00	1.00	1.00	1.00
			10	1.00	1.00	1.00	1.00	1.00	1.00

Notations: π_1: mixing proportion of the first component; n: sample size; λ_2: the second component mean.
Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 10 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.04 for each configuration.

Table B.2 (Continued) – Simulated power of the LRT for testing ZIPs using the PPC method based 42 configurations of prior parameter values (4.1) (N= 500 replicates)

				Simulated Power (Kepner and Wackerly Values)					
				t					
π_1	n	λ_2	α	0.1	0.5	1	3	5	10
0.5	100	1	1.1	0.93	0.92	0.92	0.92	0.92	0.92
			1.5	0.92	0.92	0.92	0.92	0.92	0.92
			2	0.92	0.92	0.92	0.92	0.92	0.93
			3	0.92	0.92	0.92	0.92	0.92	0.92
			4	0.92	0.92	0.92	0.93	0.92	0.92
			5	0.92	0.93	0.92	0.91	0.93	0.92
			10	0.92	0.92	0.93	0.92	0.92	0.92
		2	1.1	1.00	1.00	1.00	1.00	1.00	1.00
			1.5	1.00	1.00	1.00	1.00	1.00	1.00
			2	1.00	1.00	1.00	1.00	1.00	1.00
			3	1.00	1.00	1.00	1.00	1.00	1.00
			4	1.00	1.00	1.00	1.00	1.00	1.00
			5	1.00	1.00	1.00	1.00	1.00	1.00
			10	1.00	1.00	1.00	1.00	1.00	1.00
		3	1.1	1.00	1.00	1.00	1.00	1.00	1.00
			1.5	1.00	1.00	1.00	1.00	1.00	1.00
			2	1.00	1.00	1.00	1.00	1.00	1.00
			3	1.00	1.00	1.00	1.00	1.00	1.00
			4	1.00	1.00	1.00	1.00	1.00	1.00
			5	1.00	1.00	1.00	1.00	1.00	1.00
			10	1.00	1.00	1.00	1.00	1.00	1.00

Notations: π_1: mixing proportion of the first component; n: sample size; λ_2: the second component mean.
Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 10 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.04 for each configuration.

Table B.2 (Continued) – Simulated power of the LRT for testing ZIPs using the PPC method based 42 configurations of prior parameter values (4.1) (N= 500 replicates)

				Simulated Power (Kepner and Wackerly Values)					
				t					
π_1	n	λ_2	α	0.1	0.5	1	3	5	10
0.5	200	1	1.1	1.00	1.00	1.00	1.00	1.00	1.00
			1.5	1.00	1.00	1.00	1.00	1.00	1.00
			2	1.00	1.00	1.00	1.00	1.00	1.00
			3	1.00	1.00	1.00	1.00	1.00	1.00
			4	1.00	1.00	1.00	1.00	1.00	1.00
			5	1.00	1.00	1.00	1.00	1.00	1.00
			10	1.00	1.00	1.00	1.00	1.00	1.00
		2	1.1	1.00	1.00	1.00	1.00	1.00	1.00
			1.5	1.00	1.00	1.00	1.00	1.00	1.00
			2	1.00	1.00	1.00	1.00	1.00	1.00
			3	1.00	1.00	1.00	1.00	1.00	1.00
			4	1.00	1.00	1.00	1.00	1.00	1.00
			5	1.00	1.00	1.00	1.00	1.00	1.00
			10	1.00	1.00	1.00	1.00	1.00	1.00
		3	1.1	1.00	1.00	1.00	1.00	1.00	1.00
			1.5	1.00	1.00	1.00	1.00	1.00	1.00
			2	1.00	1.00	1.00	1.00	1.00	1.00
			3	1.00	1.00	1.00	1.00	1.00	1.00
			4	1.00	1.00	1.00	1.00	1.00	1.00
			5	1.00	1.00	1.00	1.00	1.00	1.00
			10	1.00	1.00	1.00	1.00	1.00	1.00

Notations: π_1: mixing proportion of the first component; n: sample size; λ_2: the second component mean.
Notes: 1. 500 simulated samples of size 50, 100, and 200 were used. 2. 10 random starting points were used for the EM algorithm. 3. The nominal significance level was 0.05. 4. The margin of error at 95% confidence is ±0.04 for each configuration.