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Detection Algorithm for Tracking Variable and Multiple Target Tracking by Particle Filtering

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Abstract

In most multitarget tracking approaches based on Joint Probabilistic Data Association (JPDA), it is difficult to apply the solutions to problems (due to the dimensionality curse of heavy complexity) where the number of target varies dramatically. In this paper, we introduce an Algorithm for Detection of Multitargets in Wireless Acoustic Sensor Networks (ADMAN); we localize detected targets by particle filtering after ADMAN. The purpose of ADMAN is detecting any number of targets (We know the approximate locations of targets during the detection algorithm.) in the field of interest. The advantage of ADMAN is its ability to cope with varying number of targets in time. ADMAN does not have any restrictions on the varying pattern of the target number.

I. INTRODUCTION

We use measurement or information to estimate the state of a target of which process is called tracking. Many kinds of physical quantities related with the state of the target can be used for the measurement in localization problem such as estimate of position, range and/or bearing, time of arrival difference, frequency of narrow band signal emitted by target, frequency difference due to Doppler shift sensed by two sensors, and signal strength which we employ as the measurement in this paper. Among them, Received Signal Strength (RSS) can be classified in the lowest level and does not need any signal processing to be measurement. In RSS model [1], the measured power at a sensor is expressed as follows,

\[ y_{n,t} = 10 \log_{10} \left( \sum_{k=1}^{K} \frac{\Psi_k d_0^{\alpha}}{|r_n - l_{k,t}|^{\alpha}} \right) + v_{n,t}, \quad n = 1, 2, \ldots, N \] (1)

where \( l \) is the location of a source target, \( n \) is sensor index, \( t \) is time instant, \( K \) is the number of targets, \( \Psi_k \) is the received power from the source at the reference distance \( d_0 \), \( r \) is the sensor location, \( \alpha \) is the attenuation factor (\( \alpha \geq 1 \)), \( v \) is background zero-mean Gaussian noise and \( N \) is the total number of sensors used in the field.

Tracking targets, of which number is varying, by acoustic sensors is very challenging due to the dimensionality curse. Many approaches such as Joint Probabilistic Data Association (JPDA) [2], Multiple Hypothesis Tracking (MHT) [3], Finite Set Statistics (FISST) [4] and Probability Hypothesis Density (PHD) [5] have huge size of state space for the joint distribution over target states because all these approaches are associated with JPDA (Joint Probabilistic Data Association) [2]; by using solutions based on JPDA, we can not avoid huge complexity, which makes it very impractical. Consequently, it is very difficult to apply most solutions to the problems where the number of target varies dramatically; usually the varying pattern of the target number is limited such as \(-1, 0, \) or \(1 \) [6], [7].

In this paper, we introduce an Algorithm for Detection of Multitargets in Wireless Acoustic Sensor Networks (ADMAN) in addition to a localization method based on particle filtering. After detection is accomplished by ADMAN, we adopt particle filtering [8] to estimate the locations of detected targets. The phenomenal advantage of ADMAN is its ability to cope with any time variation in the number of targets without being affected by the dimensional curse. On the other hand the possibility
of the dimensionality curse still remains due to the inherent property of the Bayesian sequential Monte Carlo method in the estimation part.

The rest of paper is composed as follows. We introduce general algorithm of ADMAN in section II. In section III, we show the steps of Generalized Likelihood Ratio Test (GLRT) that is the solution of Composite Hypothesis Testing (CHT); we use CHT to apply ADMAN to detection of multitargets. MLMC method is combined with ADMAN to complete the tracking procedure of the varying multiple number of targets in section IV. We present an simulation as an example of the algorithm in section V.

II. DETECTION ALGORITHM

We explain about general algorithm of ADMAN in this section. We begin with defining two important terms used in the algorithm.

A. Range and Threshold

1) Range: Sensors are deployed uniformly and cover the whole area of interest according to a specific range of the sensors as in Fig. 1. The range is the half of the distance between any two neighboring (horizontally or perpendicularly) sensors. We also define the range of a sensor as the distance between the “sensor” and the “boundary” of the region such that the power received by the sensor from a single target within the region is greater than a predefined threshold. We can regard the range of the sensor as the capability to resolve two targets because ADMAN will fail if there are more than one target exist within the boundary of the sensor range. The smaller the range, the better the resolution is; this implies we need more sensors to have finer resolution.

2) Threshold Power: An acoustic sensor sends the signal to the fusion center that there is a target around itself, (It does not have exact information of the target location, but certainly inside of boundary of the range.) after it makes a decision of detection according to the decision rules. We need threshold power to apply hypothesis test to the decision rules. Threshold
power ($\Delta_1$) is defined as the received power from a single target which is located on the range boundary while no noise power is considered. This constructs the first step of the ADMAN. As long as a sensor receives the power greater than the threshold, it exclusively detects a target within the boundary of the range. More complicated situation forms another step of the algorithm with introduction of another threshold power ($\Delta_2$). As shown in Fig. 2, $\Delta_2$ is defined as the received power from a single target which is located on the circle of radius $\sqrt{2}r_0$ distance. When a target is located in “in-between” area (‘In-between’ area is defined as the area that does not belong to any circle-region of the range.), this other threshold is necessary to resolve the location of target. No more than one sensor is supposed to receive power signal greater than $\Delta_1$, but more than one sensor may receive power signal between $\Delta_1$ and $\Delta_2$. So if the strength of the received power is between $\Delta_1$ and $\Delta_2$, a target is most probably located in “in-between area” of several neighboring sensors. In this case a target is attached to the sensor that has the maximum received power among those sensors. Finally the sensors do not pay attention to any received power that is less than $\Delta_2$. 

**B. Algorithm**

Now, we are ready to design the ADMAN. Then, how should we design the ADMAN to detect many targets at the same time? First of all, we have to decide the maximum number of targets that the sensor network will be designed for. Nonetheless, the exact number of targets in the field is not known. The key of ADMAN is attachment of each target to the most closest single sensor without redundancy.

If we assume that there is always only one target or none in the field of interest, it is easy to attach a target to a sensor which is the closest to the target. Regardless of the size of the range ($r_0$), the threshold is set to be the received power from a target at a distance $r_0$ from the sensor. So the sensor which has the received power signal greater than threshold will be attached to the target at any time instant. We show an example of attaching target when the maximum number is one in Fig. 3. At each time instant, the target is detected and attached to the closest sensor which received the highest power. However, a complicated situation will arise when trying to track more than one target at the same time.
Suppose that the maximum number of targets we want to track at the same time is 3, and we may want the range of the sensor to be $r_1$ (not $r_0$). If we consider the extreme situation that a sensor receives possible strongest power without detection; in this situation all three targets are located at the distance of $r_1$ from the sensor as (a) in Fig. 4. Intuitively, we could take this received power as the threshold, and if a sensor receives power greater than this threshold, it decides that there is a “single” target within the range of the sensor. However, we can find out obvious errors which are shown in Fig. 4 if we do not modify the range. With the range $r_1$ and threshold chosen previously, as shown in Fig. 4(b) and (c), even if the received power is less than threshold it makes the decision that a single target exists within the circle of the range. So we need to find another solution for these errors. With respect to each of the errors in Fig. 4, we can find the solution or adjustment as follows:

1) **In order to avoid errors as in Fig. 4 (b) or (c):** we adopt a new range “$r_0$” which is shorter than the “$r_1$”. Nevertheless, the threshold power remains the same, so in the case of Fig. 4(b) or (c), the sensor does not make the decision of detection; it becomes true with the range $r_0$ because targets will be located outside of the range boundary and received power is also less than the threshold power. These targets will be attached to the neighboring sensors. We find the range $r_0$ according to (1) such that the strength of the signal from a single target at $r_0$ and the strength of the signal from all 3 targets at $r_1$ received at a sensor must be the same as shown in Fig. 5. We call $r_1$ reference range from here on. If $d_0 = 1, \alpha = 2$, and $N = 3$, then
the following condition must be satisfied according to (1).

\[
\frac{1}{r_1^2} + \frac{1}{r_1^2} + \frac{1}{r_1^2} = \frac{1}{r_0^2}, \quad r_0 = \frac{r_1}{\sqrt{3}}
\]  

(2)

2) Once we have new range \(r_0\): we encounter other types of error as in Fig. 4(d) or (e). Fig. 4(d) shows that even though two targets are outside of the range, the sensor makes decision of detection because it receives power greater than the threshold, which is an error. In Fig. 4(e), a sensor falsely makes decision of the detection too because it receives power greater than threshold as in the case of Fig. 4(d). If we relate it to the resolution of the sensor, we can avoid errors as in 4(d) and (e) because we assume that no more than one target can exist within the reference range, \(r_1\); but if there are more than one target together within the range \(r_1\) we can not resolve it and make a detection error. That is why we also call the “reference range \(r_1\)” “resolution” of the sensor. We can not resolve more than one target within the range \(r_1\). The resolution is directly related with threshold power when the maximum number of targets is set to a constant. The resolution of a sensor does not directly rely on the range of a sensor \(r_0\). The reference range \(r_1\) increases when the maximum number of targets increases under the condition of the constant \(r_0\), which means the resolving capability of a sensor decreases as the maximum number of targets increases. The threshold increases when the maximum number of targets increases under the constant resolution of a sensor.

Generally, when we apply ADMAN, detecting more number of targets does not make the problem proportionally more difficult, but the resolution decreases if all other conditions remain the same. The advantage of the ADMAN is that it can cope with any variation in the number of targets. ADMAN shows superiority when the number of targets vary dramatically.

The ADMAN without any decision rule is summarized in the Table I.

### III. GENERALIZED LIKELIHOOD RATIO TEST (GLRT)

We apply CHT [9], where the PDFs under the hypotheses are not completely known, to every sensor to decide if each sensor will be attached to a target or not. The CHT problem is solved using GLRT in this paper. There are a number of solutions for the CHT problem, e.g., Bayesian approach, Wald test, Rao test, locally most powerful test, and so on; Bayesian approach,
TABLE I

SUMMARY OF ADMAN

Field of interest is full of deployed sensors as in Fig. 1, and each sensor is identified with number from the left bottom to the right top. Each measurement of the sensors is scanned from 1 to \(N\) (Total number of sensors in the field of interest). Given \(r_0\), \(r_1\), \(\Delta_1\), \(\Delta_2\), \(N\), and \(X\) where \(r_0\) is the range of the sensor, \(r_1\) is the reference range of the sensor, \(\Delta_1\) and \(\Delta_2\) are the thresholds \((\Delta_1 > \Delta_2)\), and \(X\) is the number of sensors in one row. At any time instant \(t\),

1) For \(n = 1, 2, \ldots, N\),

- If \(y_{n,t} > \Delta_1\), attach a target to the sensor \(n\) which means sensor \(n\) believes there is a target within its range of \(r_0\).
- If \(\Delta_2 \leq y_{n,t} \leq \Delta_1\), check whether \(y_{n,t}\) is from already attached target or not. If so, we attach a target to the sensor that has greatest power among the neighboring sensors, otherwise we attach a target to the sensor \(n\).

2) After scanning and target attaching is completed, find out all attachment redundantly in both “inside range” and “in-between area”, and remove attachment in “in-between area.”

which considers the unknown parameter as a random variable with prior density function, and GLRT method (It estimates the parameter by the maximum likelihood method and applies it to the likelihood ratio test.) are two major approaches to CHT.

GLRT is the most pertinent approach because, in ADMAN, we have to estimate parameters in both hypotheses as opposed to other solutions which require parameter estimation only in one hypothesis, and Bayesian method requires the knowledge of prior and more assumptions and multidimensional integrations. In this section GLRT related to ADMAN is explained as a solution for the CHT problem in the detection part. We estimate parameters first by ML method for the multiple hypotheses test in ADMAN algorithm.

A. Hypotheses

At each time step we have three hypotheses, \(\mathcal{H}_0\), \(\mathcal{H}_1\), and \(\mathcal{H}_3\). We combine \(\mathcal{H}_0\) and \(\mathcal{H}_1\) as \(\mathcal{H}_2\); so the first hypothesis test is composed of \(\mathcal{H}_2\) and \(\mathcal{H}_3\). If \(\mathcal{H}_2\) is selected, then we go to the next hypothesis test to decide if \(\mathcal{H}_1\) or \(\mathcal{H}_0\). Under \(\mathcal{H}_3\), we have target within the boundary circle of the sensor range, under \(\mathcal{H}_1\), we have a target in “in-between” area, and under \(\mathcal{H}_0\), some other sensor will be related and attached to the target. This two stage hypothesis testing procedure can be summarized as in Fig. 6. So the multiple hypotheses testing is modified to two steps of single hypothesis testing. Referring to (1), we can rewrite the received power of sensor \(n\) at time instant \(t\) as follows,

\[
y_{n,t} = \theta_{n,t} + v_{n,t}, \quad n = 1, 2, \ldots, N
\]

where \(\theta_{n,t} = 10 \log_{10} \left( \sum_{k=1}^{K} \frac{\Psi_{k} d_{k}^{\alpha}}{|r_n - r_{k,t}|^{\alpha}} \right)\). All the hypotheses are described as follows,

\[
\mathcal{H}_3 : \theta_{n,t} > \Delta_1
\]
Fig. 6. Procedure of decision of detection in hypotheses testing performed with respect to every sensor.

\[ H_2 : \theta_{n,t} = \theta^2_{n,t} \leq \Delta_1 \]

\[ H_1 : \theta_{n,t} = \theta^1_{n,t} \quad \Delta_2 < \theta^1_{n,t} < \Delta_1 \]

\[ H_0 : \theta_{n,t} = \theta^0_{n,t} \leq \Delta_2 \]

where \( \Delta_1 \) and \( \Delta_2 \) are threshold powers as defined previously. The decision regions and probabilities are shown in Fig.s 7 and 8. The decisions are performed by two steps as in the Table II and III. If \( H_2 \) is decided in the first test, we will go to the second test to complete decision.

Fig. 7. First hypothesis testing.

Fig. 8. Second hypothesis testing.
TABLE II

FIRST HYPOTHESIS TESTING

To perform GLRT we have to find the estimates of parameters by the maximum likelihood estimate (MLE) method as,

\[ \hat{\theta}_3 = \begin{cases} y_{n,t}, & \text{if } y_{n,t} \leq \Delta_1, \\ \Delta_1, & \text{take } \hat{\theta}_3 = \Delta_1. \end{cases} \] (4)

\[ \hat{\theta}_2 = \begin{cases} y_{n,t}, & \text{if } y_{n,t} > \Delta_1, \\ \Delta_1, & \text{take } \hat{\theta}_2 = \Delta_1. \end{cases} \] (5)

next, GLRT decides \( H_3 \) if

\[ L_G(y_{n,t}) = \frac{p(y_{n,t}; \hat{\theta}_3, H_3)}{p(y_{n,t}; \hat{\theta}_2, H_2)} > \gamma_{32} \] (6)

where with the variance of noise, \( \sigma \),

\[ p(y_{n,t}; \hat{\theta}_3, H_3) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left[ -\frac{1}{2\sigma^2} (y_{n,t} - \hat{\theta}_3)^2 \right] \]

\[ p(y_{n,t}; \hat{\theta}_2, H_3) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left[ -\frac{1}{2\sigma^2} (y_{n,t} - \hat{\theta}_2)^2 \right] \]

Then (6) also can be easily shown as,

\[ \ln L_G(y_{n,t}) = \frac{1}{2\sigma^2} (2\hat{\theta}_3y_{n,t} - 2\hat{\theta}_2y_{n,t} - \hat{\theta}_3^2 + \hat{\theta}_2^2) > \ln \gamma_{32} \] (7)

Given certain probability of false alarm, \( P_{32}^{FA} \), it is defined and related with threshold, \( \gamma_{32} \) as follows,

\[ P_{32}^{FA} = P(H_3|H_2) = P_r\{y_{n,t} > \gamma_{32}; H_2\} \]

\[ = \int_{\gamma_{32}}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left[ -\frac{1}{2\sigma^2} (t - \hat{\theta}_2)^2 \right] dt \]

\[ = Q\left(\frac{\gamma_{32} - \hat{\theta}_2}{\sigma}\right) \] (8)

IV. LOCALIZATION FOR DETECTED TARGETS

This section is devoted to the localization of the targets which are already detected by ADMAN. In the model, targets are dynamically moving according to the state space equation as follows [10],

\[ x_t = G_x x_{t-1} + G_u u_t \] (14)

where \( x_t = [\ddot{x}_1,t \hfill \ddot{x}_2,t \hfill \dot{x}_1,t \hfill \dot{x}_2,t \hfill x_1,t \hfill x_2,t]^T \) is the state vector which indicates the acceleration, velocity, and position of the target in a two-dimensional Cartesian coordinate system; \( G_x \) and \( G_u \) are known matrices defined by
After finding MLEs of $\theta_1$ and $\theta_0$ as following,

$$\hat{\theta}_1 = y_{n,t}, \quad \text{but if } y_{n,t} \leq \Delta_2, \quad \text{take } \hat{\theta}_1 = \Delta_2$$

$$\hat{\theta}_0 = y_{n,t}, \quad \text{but if } y_{n,t} > \Delta_2, \quad \text{take } \hat{\theta}_0 = \Delta_2$$

(9) (10)

the GLRT decides $H_1$ if

$$L_G(y_{n,t}) = \frac{p(y_{n,t}; \hat{\theta}_1, H_1)}{p(y_{n,t}; \hat{\theta}_0, H_0)} > \gamma_{10}$$

(11)

where with the variance of noise, $\sigma$ ,

$$p(y_{n,t}; \hat{\theta}_1, H_1) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2\sigma^2}(y_{n,t} - \hat{\theta}_1)^2\right]$$

$$p(y_{n,t}; \hat{\theta}_0, H_0) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2\sigma^2}(y_{n,t} - \hat{\theta}_0)^2\right]$$

Then (11) also can be easily shown as,

$$\ln L_G(y_{n,t}) = \frac{1}{2\sigma^2}(2\hat{\theta}_1 y_{n,t} - 2\hat{\theta}_0 y_{n,t} - \hat{\theta}_1^2 + \hat{\theta}_0^2) > \ln \gamma_{10}$$

(12)

Given certain probability of false alarm, $P_{FA}^{10}$, it is defined and related with threshold, $\gamma_{10}$ as follows,

$$P_{FA}^{10} = P(H_1|H_0) = P\{y_{n,t} > \gamma_{10}; H_0\}$$

$$= \int_{\gamma_{10}}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2\sigma^2}(t - \hat{\theta}_0)^2\right] \, dt$$

$$= Q\left(\frac{\gamma_{10} - \hat{\theta}_0}{\sigma}\right)$$

(13)

$$G_x = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
T_s & 0 & 1 & 0 & 0 \\
0 & T_s & 0 & 1 & 0 \\
\frac{T_s^2}{2} & 0 & T_s & 0 & 1 \\
0 & \frac{T_s^2}{2} & 0 & T_s & 0
\end{pmatrix}$$

and $G_xu_t = u_t = \begin{bmatrix} \omega_1 & \omega_2 & 0 & 0 & 0 \end{bmatrix}^T$.

$T_s$ is the sampling period time (s), $\omega_i$ is uniformly distributed in $[-W_{\text{max}}, W_{\text{max}}]$ (m/s$^2$). We are interested in the location of the targets, so only the location part of state will form the measurement as shown in (1). Our goal is to estimate $x_{1,t,k}$ and $x_{2,t,k}$ given $\hat{n}_{1:K,t}$, the estimated identities of sensors attached to the $K$ targets from the ADMAN. We estimate the states of the detected targets by using particle filter. If a new target appears in any sampled time instant, we generate uniform particles
on the area which is decided by ADMAN. Continuing targets are estimated by sequentially generated particles. The localization by the particle filtering is summarized in the Table IV.

V. SIMULATION

We present an example of tracking variable number of targets by ADMAN and particle filtering. All independent targets are dynamically moving according to Markov Chain dynamic state model specified in the previous section. The maximum number of targets is 3 and initially two targets are moving in the filed of interest. The other target appears spontaneously in 4 sampling time instants later. The first target lasts 50 time steps and disappears, the second target lasts 46 time steps and disappears, the last one lasts 4 time steps more after first target disappears and 8 time steps more after the second target disappears.

The sensors are uniformly deployed on two dimensional rectangular area of interest. The reference range, the $r_1 = 30 (m)$, therefore the range, $r_0 = \frac{r_1}{\sqrt{3}} = 17.3205 (m)$. The threshold powers, $\Delta_1 = 10000/r_0^2 = 33.33 (J/s)=15.2288 (dB)$ and $\Delta_2 = 10000/(\sqrt{2}r_0)^2 = 16.6667 (J/s)=12.2185 (dB)$. $W_{max}$ is equal to 0.4 $(m/s^2)$, $\Psi_1 = \Psi_2 = \Psi_3 = 4 (dB)$, $d_0 = 1 (m)$, $\alpha = 2$, $v_{n,t}$ is Gaussian noise with zero mean and variance of 0.03 $(J/s)$, $P_{FA}^{32} = P_{FA}^{10} = 10^{-3}$ and the number of particles is 1500. The starting points of each targets are (200,200), (0,100) and (200,50). After deploying the sensors, we give identifications to all sensors with numbers from the left bottom to the right top. According to ADMAN, any detected targets will be attached to a specific sensors identified by its number exclusively. The simulation shows the result of detection of targets with time in Fig. 9 and the estimation of the tracked locations of the targets in Fig. 10.
VI. CONCLUSIONS AND FUTURE WORK

We proposed ADMAN for detecting unknown and varying number of targets with highly non-linear motion in a two dimensional rectangular region of acoustic sensor networks. When applying ADMAN, GLRT is adopted for the solution of composite hypothesis testing. After any target is detected, particle filter is employed to estimate the locations of targets. Experiments showed the ADMAN is robust and accurate in multitarget tracking problem where unknown number of targets keep on varying. The algorithm efficiently handles more than 2 targets even though the measurement of acoustic sensor is almost the same as raw data. In this paper the information of the initial location of any target is not known in accord with practical situation as opposed to Bayesian models (Even though the particle filter is based on Bayesian frame). The influences of the changing parameters, e.g., the range $r_0$, threshold power, $\Delta_1$, and $\Delta_2$ will be investigated in the future. The proposed approaches can alleviate the computational complexity of solving problems of tracking unknown and varying multitargets, especially in detection problems in acoustic wireless sensor networks. Even if a number of targets are to be tracked, the increased computational complexity of the ADMAN is not remarkable at all. If we want to resolve multiple targets when they are very close, we can increase the resolution of the sensor at the expense of the increased number of sensors deployed in the field of interest. When we have more sensors in the field of interest, the range can be much smaller, which mainly decides the resolution of the sensors.

REFERENCES


After detection step is accomplished we have estimated sensors attached to \( n_2 \) targets as \( \hat{n}_{1:n_2,t} \). At any time instant \( t \), if \( n_1 \) is the number of targets in previous sampling time instant, \( \rho \) is the number of particles,

(1) Compare \( n_1 \) and \( n_2 \)

- Case \( n_2 = n_1 \)
  a) Generate sequential particles of locations from the previous sampling time sequence. FOR \( i = 1 : \rho \), \( l_{1:n_2,t}^i \sim p(l_{1:n_2,t}^i | l_{1:n_1,t-1}^i) \)
  b) Compute weights of particles. FOR \( i = 1 : \rho \), \( F_{y_t}^i = p(Y_t | l_{1:n_2,t}^i) \), normalize it to \( \tilde{F}_{y_t}^i \).

- Case \( n_2 < n_1 \)
  a) Generate \( C_{n_1}^{n_2} \) combinations of tracks of particles. FOR \( j = 1 : C_{n_1}^{n_2} \), \( l_{1:n_2,t}^{1:\rho,j} \sim p(l_{1:n_2,t}^i | l_{1:n_1,t-1}^i) \)
  b) Compute likelihood functions. FOR \( j = 1 : C_{n_1}^{n_2} \), \( F_{y_t}^{1:\rho,j} = p(Y_t | l_{1:n_2,t}^{1:\rho,j}) \)
  c) Choose the best track.

\[
\tilde{l}_{1:n_2,t}^i = \max_{l_{1:n_2,t}^{1:\rho,j}} F_{y_t}^{1:\rho,j}
\]

- Case \( n_2 > n_1 \)
  a) Generate particles continuously with \( n_1 \) targets. FOR \( i = 1 : \rho \), \( l_{1:n_1,t}^i \sim p(l_{1:n_1,t}^i | l_{1:n_1,t-1}^i) \)
  b) Generate new particles with \((n_2 - n_1)\) targets uniformly distributed according to decision of detection hypothesis test which forms \( C_{n_2-n_1}^{n_2} \) combinations of tracks combined with continuous targets. FOR \( j = 1 : C_{n_2-n_1}^{n_2} \), FOR \( k = 1 : (n_2 - n_1) \)

- Case \( H_3 \)

\[
l_{k,t}^{1:\rho,j} \sim U(r_{k,t}^{j} + r_0 \hat{n}_{1:n_2,t})
\]

- Case \( H_1 \)

\[
l_{k,t}^{1:\rho,j} \sim U(r_{k,t}^{j} + r_0, r_{k,t}^{j} + \sqrt{2}r_0 \hat{n}_{1:n_2,t})
\]

- Combine sequential particles and new particles,

\[
l_{1:n_2,t}^{1:\rho,j} = (l_{1:n_2-n_1,n_2-1}^{1:\rho,j}, l_{1:n_1,t}^{1:\rho})
\]

- Compute likelihood functions. FOR \( j = 1 : C_{n_2-n_1}^{n_2} \), \( F_{y_t}^{1:\rho,j} = p(Y_t | l_{1:n_2,t}^{1:\rho,j}, \hat{n}_{1:n_2,t}) \)

- Choose the best track.

\[
\tilde{l}_{1:n_2,t}^i = \max_{l_{1:n_2,t}^{1:\rho,j}} F_{y_t}^{1:\rho,j}
\]

- Normalize chosen weight to \( \tilde{F}_{y_t}^i \)

(2) Estimate location of targets. \( \hat{l}_{1:n_2,t} = \sum_{i=1}^{\rho} \tilde{l}_{1:n_2,t}^i \tilde{F}_{y_t}^i \)

(3) Resample \( \tilde{l}_{1:n_2,t}^i \)

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**TABLE IV**

**SUMMARY OF LOCALIZATION BY PARTICLE FILTERING**