High School Mathematics Course Offerings

Graduate Research Project:
High School Mathematics Course Additions

By
Mary B. DeMann

Submitted in Partial Fulfillment
of the Requirements for the Degree of
Master of Science in Mathematics Education

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled, “High School Mathematics Course Offerings” submitted by Mary B. DeMann in partial fulfillment of the requirements of the degree of Master of Science of Mathematics Education.

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____________________________________________
Mary B. DeMann
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Creating a Vision

In their *Principles and Standards for School Mathematics*, the National Council of Teachers of Mathematics presents a vision for mathematics. This vision includes “ambitious expectations for all, with accommodation for those who need it. Knowledgeable teachers have adequate resources to support their work and are continually growing as professionals.” (Principles and Standards for School Mathematics Appendix 2000). Mathematically rich curricula is part of this vision. Students who value Mathematics and are actively engaged learners of mathematics are also part of this vision. This research will focus on curricula, and also student interest and appreciation of mathematics.

Successful curricula include a desire to convey mathematics as a personally satisfying and empowering life-tool. As master teachers, we have the opportunity and responsibility to create mathematics in the classroom that focuses on developing a life-long appreciation and interest for the subject. We also have the obligation to help students fulfill their New York State graduation requirements in mathematics (NYSUT.org/standards/students_graduation.html). The master teacher incorporates both of these into classroom practices that work together to achieve rich, successful curricula.

Secondary school students face many challenges that develop each student individually. Students should view each of their four years of high school as an opportunity to develop and strengthen their mathematical skills
and ability. Upper-level courses can and should further develop mathematical understanding and mastery as students apply them to the real world. For instance, the use of exponential functions to model exponential growth in microbiology labs and pharmaceutical research, or linear, quadratic, cubic functions (and the use of technology to find these functions) and rates of change to model the physics found in an amusement park (acceleration, velocity, slope, and pendulum motion) are various examples of applying student understanding and mastery of mathematics.

It was believed that WPHS students lacked interest in and appreciation for mathematics as a life-long skill. The WPHS Mathematics Department also believed that the lack of variety of upper-level mathematics courses being offered hindered its ability to meet student needs. Finally, it was found that the current Math 12 curriculum did not properly prepare students for AP Calculus.

This research project is devoted to researching student needs, interests, and level of appreciation for mathematics. It further includes developing curriculum for both current courses and possible new courses, and incorporating NCTM Principles and Standards, and New York State Standards for mathematics so that this vision can be achieved. This research project has assisted Whitney Point High School (WPHS) Mathematics Department in moving closer to reaching this vision.
Current offerings at Whitney Point

Historically, WPHS has offered two courses beyond the Mathematics B material – AP Calculus and Introduction to Statistics. The WP administration made a decision several years ago to offer the Mathematics B Regents exam in January of the fourth year of Mathematics. The Mathematics B exam originally included many topics beyond the traditional Course III material. More recent exams are structured closer to the Course III material, so the current WP administration made a decision to administer the exam in June after the third year of mathematics. This decision allowed the WP Mathematics Department to develop new course material for the Mathematics 12 course.

As the survey was developed, more questions arose within the Mathematics Department. A gap was found in the material between Mathematics 12 course and AP Calculus. The current Mathematics 12 course was created several years ago. In 1991, New York State developed a compendium of topics for Precalculus that contained about two years of material. Teachers were advised to select material from the Precalculus topics, and create a one-year course based on that material. This led to vast differences in Mathematics 12 courses across the state. This has been a topic of discussion for some time in the WPHS Mathematics department, but it was not able to address the issue until now.
The current offerings at WPHS are:

**Mathematics 12**  Topics include Real Number Properties, Complex Numbers, Rectangular Coordinates, Systems of equations and inequalities, Relations, Functions, Rational Functions, Exponents and Exponential Functions, Logarithmic Functions, Trigonometric Identities, Normal curve, Binomial theorem and probabilities.

**Statistics**  This is a ½ year introductory course that includes an introduction to Statistical Methods, Probability Distributions, Normal Probability Distributions, Estimates and Sample Sizes.

**AP Calculus**  This course includes a numerical, graphical, and analytical approach to Calculus.  Topics include Limits, Derivatives, Derivative Applications, Riemann Sums, Integrals, and Applications of Integrals.

Current Accelerated Program at Whitney Point:

Like many other local school districts, Whitney Point offers an accelerated mathematics program beginning at the 8th grade level.  These accelerated students are homogeneously grouped in their 8th grade class and are instructed in the first year of high school mathematics (Algebra).  As Freshmen in high school, they are heterogeneously grouped for the remaining three years of traditional mathematics instruction.  The WPHS has averaged approximately forty accelerated students per year.  Over the past several years, WPHS has noticed that many of these accelerated students choose not to take mathematics in their senior year of high school, even though the majority of the accelerated
students plan on attending college when they graduate. It is believed that this weakens their mathematics skills needed in college by losing a year of mathematics instruction. Also, these students will be at a disadvantage among their college peers during their first year of college, since their mathematics skills are not as strong as their peers.

Local area guidance counselors and teachers gave information that suggested that upper-level courses such as Statistics, College Algebra, Mathematics Methods, and Calculus I are generally well attended and are beneficial to the preparation of college-bound students. Many high schools also offer college credit through community colleges. Students who do not plan on attending college also attend these classes.

The WPHS Mathematics Department has several offerings for non-college track students. These classes are designed to prepare students for the New York State minimum graduation requirements, as well as to develop interest and abilities. These classes move at a manageable pace and offer projects, computer based investigations, and integrated topics, so student interest and involvement stays high. For many years, the WPHS Mathematics Department has felt obligated to spend its energy and attention on helping the non-college-bound student meet graduation requirements as creatively as possible, and has done a fine job meeting non-college-bound student needs. As the department continued to self-evaluate it’s processes, it has decided to focus on upper-level offerings to aid college bound students.
Possible Additional Offerings

**College Statistics** – This will be a full year college-level Statistics course focusing on Probability, Estimates and Sample Sizes, Hypothesis Testing, Inferences, Correlation and Regression, and Analysis of Variance.

**College Algebra** – This will be a full year college-level Algebra course covering topics such as Functions and Relations, Exponential, Logarithmic Functions, Sequences and Series, and Conic Sections.

**College Trigonometry** – This will be a full year college-level Trigonometry course focusing on Trigonometric Functions, Circular Functions, Vectors, Complex Numbers, and Polar and Parametric Equations, Trigonometric or Polar Form of Complex Numbers, and Operations.

**Algebra and Geometry** – This course will preview advanced Mathematics that do not require calculus. The focus will be on the connection between algebra and geometry, including Analysis of Functions and Relations, Quadratic Functions and Inequalities, and Inverse Relations.

**College Prep Calculus** – This will be a full year Calculus program designed to prepare students for further College-level Calculus. The topics will include graphical, numerical and analytical analysis of functions, including Logarithmic, Exponential, Trigonometric, and Rational functions. We will also develop the meaning of and application of the derivative in terms of slope of the tangent line, rate of change and local linear approximation. This course will introduce students to calculus concepts, while maintaining a less stressful pace than the
AP Calculus that is currently offered, so that students will not become frustrated. The class will also include a variety of projects to allow students the opportunity to apply and demonstrate their knowledge of the material (Syllabus is found in Appendix C and sample unit is found in Appendix D).

Some of these courses could be offered as a fourth year of high school mathematics alongside the Mathematics 12 or half year Statistics, or in place of these courses. WPHS’s goal is to promote mathematical subject matter interest while maintaining relevant course material so that students will desire to continue their high school mathematics education.

The Mathematics Department at WPHS has committed to the addition of an upper level mathematics course. Once a specific course has been selected, the Department will work as a team to design a course structured around the needs of WPHS students. This team will work together to create a course description, prerequisites (possibly adjusting current courses), and course syllabus.

The current Mathematics 12 course focuses on completing the Mathematics B curriculum and administering the exam in January. The course emphasis after that is a selection of Mathematics 12 topics such as:

- Relations, Functions
- Systems of equations and Inequalities
- Exponential functions, growth and decay
- Polynomial and Rational Functions
- The Nature of Graphs
• The Trigonometric Functions, Graphs
• Trigonometric Identities and Equations

Although these topics fall under Mathematics 12 curriculum (NYS Core Curriculum Algebra 2 and Trigonometry strands), they are not strictly focused on Precalculus, and are not sufficient to precede a Calculus course. It was recommended to the department that the current Mathematics 12 curriculum be redeveloped to follow Precalculus curriculum that includes limits, the tangent line problem, and some derivatives and applications. This will address the gap between Mathematics 12 and Calculus.
Research Process Overview

The research began in discussion style questions and ideas about current offerings at Whitney Point. The researcher then developed a plan to gain more information by surveying students, graduates, and colleagues. Surveys were developed, administered, and analyzed. The researcher sought more information from college web sites (List of websites is found in the references), and developed recommendations. The WPHS Mathematics Department made decisions based on these recommendations, and the process of curriculum development began.

The data source used included:

1. WPHS seniors, juniors, and sophomores. These students have recently made the decision to include mathematics in their senior year or not, and will add their understanding of the impact of their decision.

2. Mathematics Teachers at local high schools. Their responses included what offerings they have at their high school, and what solutions they have found successful.

3. WPHS graduates who are currently completing four years of college instruction. These students will be able to directly address the impact of their high school mathematics instruction.

4. WPHS graduates who have completed four years of college instruction. These students have a well-rounded insight and will add maturity to the discussion.
5. College Web Sites. This source was used in determining what degree program requirements were for mathematics. This was a basis to determine what emphasis students should be placing on their mathematics education.
Processes and Methodology

The data were collected from written surveys, interviews, and web sites. Once collected, the data were analyzed using qualitative categories, and some descriptive statistics. The surveys included multiple choice questions, short answer, and comment sections (see Appendix A), allowing the interviewee every opportunity to fully express themselves concerning the question. Current WPHS students (one hundred twenty-five) were surveyed during their mathematics class, and all surveys were completed and handed in. All but one of these surveys were used. The graduating class of 2001 was surveyed (one hundred thirty-two), and seventy-seven graduates responded. Thirteen area high schools were represented in the response from the teacher surveys.

One hundred twenty-five students from our Course III and Mathematics 12 classes were surveyed, following Whitney Point administration guidelines for student surveys. Forty-two students were accelerated sophomores. Thirteen New York State school districts were represented in the teacher surveys. The surveys were collected through the AMTNYS list serve forum, and were open to anyone. Each member of the graduating class of 2001 received a questionnaire. The tabulated results were analyzed statistically to reveal additional information about the results. The final phase summarizes the entire study. The survey results were presented and analyzed, and the recommendations are based on these results (Results found in Appendix A, following Student Survey.)
Originally several college mathematics professors were to be interviewed, but time was of the essence for the findings to be useful, so it was decided instead to investigate the mathematics requirements for a variety of career choices at several colleges. The researcher contacted several area guidance counselors to gain their advice. They informed the researcher on the most popular career choices for college bound students, as well as the various schools students would be most likely to attend. Among the schools were:

- Broome Community College
- Cornell University
- State University of New York at Cortland
- State University of New York at Binghamton
- University of Buffalo

These were a representative group of colleges based on the career and college choices of a sampling of graduates, as well as input from many area high school guidance departments. Some of the more popular career choices were:

- Nursing
- Business Administration
- Physical Therapy
- Engineering

Mathematics requirements for the degree programs for these career choices were the information required for the research. While many career choices involve high levels of continued mathematics education, most career choices involve at least two semesters of mathematics. These findings were not a
surprise. The majority of colleges have a type of Calculus (business calculus, Calculus I) as a base for most career choices. Many also have a type of Statistics course, while the third most required course was a type of advanced Algebra. The recommendations that follow are based on this data.
Results

As predicted, a noticeable gap in mathematics instruction was found, particularly with accelerated students who chose not to take a mathematics course in their senior year. A lack of student awareness of mathematics requirements based on their career choices was also noticed. A solution is to increase student awareness of this problem so that they base future decisions on the possible impact to them. A better understanding of the student’s viewpoint of their mathematics instruction in regards to its usefulness, enjoyment, and interest will help.

Of the graduates surveyed, only 9% felt that they were less prepared for college mathematics than their fellow college freshmen. Even though the students did not feel less prepared, after having conversation with many college professors, they indicated that freshmen are being inadequately prepared for college course work in general, mathematics being no exception.

Of the HS students surveyed:

- 90% plan on attending college.
- 61% surveyed know what their college major will be.
- 52% believe that mathematics is required for their college major.
- 37% enjoy mathematics.
- 17% chose mathematics as their favorite subject.
- 45% believe they have strong mathematics skills.
- 78% plan to take a 5th year of high school mathematics.
- 82% plan to take mathematics in their senior year of high school.
More than \( \frac{1}{2} \) of the HS students surveyed were unclear of the mathematics component for graduation requirements for NYS Regents. Almost 85% had no idea of the mathematics required for their college major.

Post-graduate students surveyed:
The post-graduate students surveyed were the entire class of 2001. Results of previous class surveys were also used as a base, and to address any discrepancies in the results.

- 87% of the respondents attended college.
- 38% of the respondents attended a 4-year college.
- 26% felt that they were better prepared than their fellow college freshmen.
- 65% felt that they were prepared equally with their fellow college freshmen.
- 9% felt that they were less prepared than their fellow college freshmen.
- 76% felt that they were adequately prepared for college mathematics.
Student Interest

• This chart displays student interest in additional math courses. The current courses as well as the possible additional courses are listed. Many students expressed interest in more than one upper-level course.

College Aspirations

• This chart displays the number of years of college that the surveyed students believe they will need in order to fulfill the requirements of their career choices. As expected, many HS students surveyed have not yet chosen a career. Most, however, were decided upon the number of years they believed they would attend college. The survey did not distinguish between graduates who will transfer from a 2-year school to a 4-year school.
Years of HS Math

• This chart displays the number of years of high school math that students plan on taking. Remember, 90% of the students surveyed plan on attending college. Interestingly, several students who said that they would take 4 years of HS math also said that they would complete AP Calculus, which is a 5th year course.

Of the High School teachers surveyed:

• 85% of the respondents teach upper-level mathematics courses.
• 38% were not satisfied with their current mathematics program.
• 69% feel that their district offers several upper-level courses.
• 46% feel that their mathematics program is dictated by budget and size.
• 38% do not feel that their upper-level courses are well attended.
• Most teachers felt that they were actively involved in the decision-making process at their school.
• Most teachers believed that more than half of their student body was college bound.
• Varying percentages in response to the number of students who choose to “opt out” of additional mathematics in their senior year.
Recommendations

This research revealed a variety of problem areas that require attention, revitalization, and focus. Among these are:

1. A noticeable gap in the sequence of the upper-level mathematics courses at Whitney Point.
2. Student interest and appreciation for mathematical subject matter.
3. Student ignorance of college mathematics requirements.
4. Importance of teacher input in administrative decisions.

To address the gap in sequence between Course III and Mathematics 12, it is recommended that WPHS Mathematics Department develop new curricula to re-align the Math 12 course to the NCTM Principles and Standards and New York State Standards of Mathematics, so that it will be a Precalculus course. Because the WPHS administration decided to administer the NYS Regents Mathematics B exam in January of the fourth year of high school mathematics, the current curriculum is designed to prepare students for the material found on the Mathematics B exam. This poses a problem with the curriculum, since it does not allow time to develop true Precalculus topics. The current Whitney Point administration has decided that it will administer the Mathematics B exam in June of the third year of mathematics. This is a more traditional approach, and fits nicely with NYS Board of Regents decision to phase out the mathematics A and B exams and replace them with annual exams.

The second area requiring attention, student interest in mathematics, will be addressed in curriculum development and re-aligning process. Included in
the appendices are several lesson plans that focus on demonstrating the usefulness of mathematics in real-world applications, and that focus on interesting, involving, and discovery-type presentation. These new lessons will inspire true mathematics appreciation and interest for mathematics.

An exciting addition to the mathematics program was a field trip to Dorney Park. Several years ago, the Physics students took an annual field trip to Darien Lake to investigate the application of Physics at amusement parks (acceleration, force, momentum...). This trip was removed from the school budget some time ago. This year, however, the Mathematics Department at Whitney Point High School asked the administration for approval to take the Course III and AP Calculus classes to “Coaster Quest at Dorney Park”. Coaster Quest is a project designed to investigate the mathematics found in amusement park rides such as roller coasters. This trip inspired much interest in mathematics throughout the entire building! The trip to Dorney Park also incorporated “IMOVIE”, a software program designed to aid in documentary productions. Using footage filmed from the Dorney Park trip, an IMOVIE was produced. The IMOVIE was shown to the entire student body at an incentive and awards assembly. The “mathematics buzz” among the students was tremendous! Students were excited about the trip, and rediscovered an interest and appreciation for mathematics. This trip is an example of the type of curricula WPHS Mathematics Department envisions for its students, and the WPHS Mathematics Department is keeping this trip as a priority in focus.
The last two findings are also important. Both the lack of student knowledge of college requirements, and the importance of teacher input in administrative decisions are valuable findings, but are beyond the scope of this research project. This concern has been discussed with the WPHS guidance department. The Mathematics Department at Whitney Point feels fortunate to have a current administration that listens to and acts upon the recommendations of the Mathematics Department for the mathematics program.

Based on both college requirements degree programs for student career choices, it was recommended to the department to add a Calculus I course. This course will not be as rigorous as the current AP Calculus course, and will allow the opportunity for creative investigation of the usefulness of several calculus applications. It was also recommended to the department that the current Mathematics 12 course be redeveloped so that it is re-aligned with current NCTM standards and principles. The research surveys indicated a weakness within the department to creatively develop interest for the subject matter, but the scope of this current recommendation will only apply to the upper-level courses.
Summary

The WPHS Mathematics department has agreed to the recommendation to develop the Mathematics 12 course as a Precalculus course. This involved investigating the current Course III curriculum to see what material was covered so that no gaps were left and no overlaps occurred in developing the new mathematics 12 (Precalculus) curriculum. The new Precalculus syllabus is included as part of this project in Appendix C. The curriculum development process included post-survey questions to local Precalculus teachers to determine what topics were focused on. College Precalculus syllabi were also used to determine what the course should include so that it would be a true Precalculus course. New texts were examined and chosen, and the curriculum was developed based on NCTM standards and the vision to maintain and develop interest and appreciation for mathematics as a life-long tool. The Mathematics Department also agreed to the addition of a Calculus I course, and this curriculum is being developed. A sample unit of lesson plans is included in Appendix C.

The Mathematics Department will also be adding an Advanced Algebra course that will be offered for a half year of the school year. Students will now have the option to take a half year of Statistics and a half year of Advanced Algebra. A course syllabus and curriculum are presently being developed by a Whitney Point Mathematics Department member.

The Mathematics Department also investigated the possibility of its students receiving local college credit for some of its courses. Students who take the
Precalculus, Calculus I, AP Calculus, Statistics, or Advanced Algebra courses will all be receiving credit through Tompkins County Community College.

This Research project was useful in many ways. The Mathematics Department worked together to evaluate its current program. Using the research, it identified some areas that needed to be addressed, and the Mathematics Department allowed the opportunity to do research that would help guide, focus, and direct its action. The WPHS Mathematics program will definitely be improved as these recommendations are implemented. Several students (seniors) have expressed disappointment at not being given the opportunity to take Calculus I (this course is listed as “College Preparation Mathematics”), or Advanced Algebra. The students have also expressed disappointment in not receiving college credit for Precalculus. While their disappointment is understandable, it is pleasing to hear their discussion among their peers. These additions and changes have inspired students to invest more time into their mathematics education. Increased class enrollment for next year supports this finding. Both the AP Calculus class and the College Preparation Mathematics class have enrollments of twenty or more students.

Another area that has seen some indirect success is increased student awareness of college requirements. The addition of college-credit courses has inspired many questions from students to the Mathematics Department and to the guidance department, and the more serious students are doing their own online research. The Whitney Point Guidance Department is aware of this issue and is seeking to improve student awareness. While the full benefits of this research project have not yet been realized, these findings are important and worth noting.
Appendix A

Student Survey

High School Student Survey
Name: _______________________________ Grade: _______________________

Strongly Agree (5)  Agree (4) Somewhat agree (3)  Do not agree (2)  Does not apply (1)

To what degree do you agree with each of the following statements?

1. I plan on attending college  ________
2. I know what my college major will be.  ________
3. Mathematics is required for my college major.  ________
4. I enjoy mathematics.  ________
5. I believe that mathematics is useful in many careers.  ________
6. I would not take mathematics if it were not required.  ________
7. Mathematics is my favorite subject.  ________
8. Mathematics develops reasoning / problem solving skills.  ________
9. I have strong mathematics skills.  ________
10. I plan on taking AP Calculus.  ________
11. I plan on taking statistics in high school.  ________
12. I would take a 5th year of hs mathematics for college credit.  ________
13. I would take Calculus in hs for college credit.  ________
14. I would take Calculus in hs for college credit.  ________

1. What is your college major? __________________________________________
   _____________________________________________________________________

2. What are your mathematics (High school) graduation requirements?
   _____________________________________________________________________

3. How many years of mathematics will you take in high school?
   _____________________________________________________________________

4. What level of mathematics will you complete in high school?
   _____________________________________________________________________

5. Will you take a mathematics course in your senior year?
6. If you answered, “yes” to (5), what mathematics course will you take?

7. What high school mathematics course would you take in hs if it were offered?

8. Would you take a mathematics course if it were offered for college credit?

9. What college mathematics courses are required for your career choice?

10. Do you feel that your mathematics ability is being fully developed?

11. Has your hs mathematics education prepared you for college?
Appendix B
High School Teacher Survey

Name Of High School: ___________________ Teacher: __________________________
Courses you teach: __________________________

Strongly Agree (5) Agree (4) Somewhat agree (3) Do Not agree (2) Does not apply (1)

Questions (1 – 5) To what degree do you agree with each of the following statements

1. I am satisfied with our current mathematics program. _______
   Additional Comments: ___________________________________________

2. We offer several upper-level (beyond Mathematics B curriculum) mathematics courses.
   Course Offerings: __________________________

3. Our students are well prepared for college mathematics courses. _______
   ___________________________________________

4. Our mathematics program is dictated by our district size and budget. _______
   ___________________________________________

5. Our upper-level mathematics courses are well attended. _______
   ___________________________________________

6. Approximately how many accelerated mathematics students take additional coursework beyond the Mathematics B assessment? (please list percent and course taken for all courses.
   a. _____% take __________________________
   b. _____% take __________________________
   c. _____% take __________________________
   d. _____% take __________________________

7. What percent of your student body is two-year college bound? _______

8. What percent of your student body is 4+ year college bound? _______

29
9. Are you as a teacher actively involved in the decision making process for the mathematics programs in your high school? 
________
Explain_______________________________________________________

10. Approximately what % of college-bound students opt out of mathematics coursework after meeting NYS graduation requirements?
____________________________________

For each upper-level mathematics course your district offers, please list:
<table>
<thead>
<tr>
<th>Course Name</th>
<th>Major Topics covered</th>
<th>Book used for college credit</th>
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Appendix C

Course Curriculum and Syllabi

Whitney Point High School
MATHEMATICS135 (Precalculus Mathematics)

High School Teacher: Teresa Delgado, Mary DeMann, Timothy Jenny

Semester(s): Full Year

Course Description:
This course provides the numerical and graphical foundation for a college calculus. An outline of the topics includes:

Relations and Functions,           One-to-one and Inverse Functions,
Domain, Range, interval notation,  Properties of logarithms and exponents,
Complex Numbers, Vectors          Amplitude, Period, and Phase shift,
The Nature of Graphs,              Graphs of Inverses,
Trigonometric Graphs,             Vertical and horizontal asymptotes,
Rational functions,               The First Derivative
Limits, Continuity of functions,  Rational, radical expressions and equations.

Prerequisites: Three years of NYS regent’s Mathematics (or equivalent high school Mathematics).

Required Text(s): PRECALCULUS WITH LIMITS – A graphing approach 4th Edition Larson, Hostetler, Edwards (McDougal Littel),2005


Required Supplement(s): TI-83 or TI-84 graphing calculator or comparable graphing calculator (NCTM Technology Standard).
Course Objectives: Students will develop problem-solving techniques using equations, functions, and graphs. Students will perform operations including logarithmic and exponential properties, and analyze graphs of various functions. Students will also find vertical and horizontal asymptotes of functions, and investigate limits and continuity. Students will be introduced to the first derivative.

Students will:

- Apply a graphical approach to problem solving. (NCTM Representation Standard)
- Investigate the nature of graphs. (NCTM Geometry Standard)
- Perform operations with logarithmic and exponential equations. (NYS Algebra Strands)
- Develop curve-sketching techniques. (NCTM Geometry Standard)
- Identify areas of discontinuity caused by holes, asymptotes, gaps and jumps. (NYS Representation Strands)
- Be introduced to limits, continuity, and the first derivative.

Attendance Policy: Attendance is required. Students are responsible for missed class work, notes, and assignments.

Grading Scale: Tests: 45%, Quizzes: 30%, Homework and class work: 25%.

Final Grade Determined by (include percentages): The 4 – 10 week marking periods are averaged and weighted 75% and the final exam is 25%.
Week 1  Real Number Properties

Week 2  Properties of Exponents, Logarithms
Solving Exponential and Logarithmic Equations

Week 3  Graphs of Logarithms
Models for Growth and Decay

Week 4  Complex Numbers
Solving Equations with Complex Numbers

Week 5  Relations and Functions
Domain and Range

Week 6  Graphs of Functions, Analyzing Graphs of Functions
Horizontal, Vertical shifts, Reflecting and Stretching Graphs
Transformations

Week 7  Quadratic Functions, Parabolas, and Problem Solving

Week 8  Algebra of Functions
Composite Functions
One-to-One and Inverse Functions

Week 9  Mathematical Modeling

Week 10  Bernoulli’s Theorem
Binomial Theorem, Binomial Coefficients

Week 11 – 12  Trigonometric Functions
Angles and their measure
Radian and Degree Measure
Right Triangle Trigonometry

Week 13  Using Trig Identities
Verifying Trig Identities

Week 14  Sum, Difference, and Double-Angle Identities
Solving Trig Equations

Week 15  Graphs of Trig Functions
Amplitude, Period, and Phase Shift

Week 16  Inverse Trig Functions
Domain, Range of Inverse Trig Functions
Week 17  Right Triangle Trig  
          Solving Inverse Trig Equations  

Week 18  Trigonometry and Complex Numbers  
          Vectors  

Week 19 - 20  Review  
               Midterm  

Week 21  Polynomial Functions  
          Locating Zeroes of Functions  
          Synthetic Division  

Week 22 - 23  Rational Functions  
               Vertical Asymptotes, holes in functions  

Week 24 - 25  Limits and Discontinuity  
               Finding Limits Graphically  
               Finding Limits Numerically  

Week 26  Continuity  
          One-Sided Limits  
          Infinite Limits  

Week 27  Trigonometric Limits  
          Squeeze Theorem  

Week 28  Average Rate of Change  
          Tangent Line Problem  
          Limit Definition of the Derivative  

Week 29 - 30  Differentiation Rules  
               Sum and Difference, Constant Multiple  
               Product Rule  

Week 31 - 32  Quotient Rule  
               Chain Rule  

Week 33  Trig Derivatives  

Week 34  Implicit Differentiation  

Week 35  Extrema on an Interval  
          Absolute extrema  

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<table>
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<tr>
<th>Week 36</th>
<th>Local Extrema</th>
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<td>Week 37</td>
<td>Optimization Problems</td>
</tr>
<tr>
<td>Week 38</td>
<td>Review</td>
</tr>
<tr>
<td>Week 39</td>
<td>Final</td>
</tr>
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Course Outline Calculus I (College Preparation Mathematics)

Course Objectives: Students will use a graphical, analytical, and numerical approach as they are introduced to limits, Derivatives, and Integral Calculus. Students will analyze graphs of various functions, and apply concepts of limits and derivatives.

Students will:

- Apply a graphical, analytical, and numerical approach to limits. (NCTM Representation Standard)
- Perform operations using the first derivative. (NCTM Algebra Standard)
- Investigate the nature of graphs. (NCTM Geometry Standard)
- Develop curve-sketching techniques. (NCTM Geometry Standard)
- Be introduced to integral Calculus, Riemann Sums, and Area under a curve. (NYS Representation Strands).

Weekly listing of reading assignments, tests, labs, projects, including due dates:

Course Content (40 weeks of instruction)

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Appendix D
Curve Sketching Unit

Calculus I
Extrema on an Interval

Curve sketching techniques display the beauty and necessity of the combination of Algebra, Calculus, and Geometry. In this unit we will form guidelines involving the application of Algebra and Calculus to determine important data used to sketch (either by hand or using a graphing utility) graphs on the coordinate plane. Graphs are a geometric representation used for real-world applications such as cost evaluation, electrical and civil engineering, physics, biology, and the list goes on and on! This unit begins with Algebra techniques to discover properties of functions.

Objectives:
1. Students will understand the definition of extrema of a function on an interval.
2. Students will be introduced to the concept of relative extrema on an open interval.
3. Students will find absolute extrema on a closed interval.

NCTM Principles and Standards
Representation: Students should be able to create and interpret models of more-complex phenomena, drawn from a wider range of contexts, by identifying essential features of a situation and by finding representations that capture mathematical relationships among those features. (Principles and Standards for School Mathematics Appendix 2000)

NYS Learning Standards for Mathematics
Algebra Strands: Students will recognize, use, and represent algebraically patterns, relations, and functions.
Geometry Strands: Students will use visualization and spatial reasoning to analyze characteristics and properties of geometric shapes. (Mathematics Core Curriculum MST Standard 3 Revised March 2005)

Instruction:

Much information can be found and used by examining the graph of a function. For instance, the maximum and minimum values of a function on a given interval are used frequently in real-life applications to determine costs, benefits, profit, etc.
Let $f$ be defined on an interval $I$ containing $c$.

1. $f(c)$ is the minimum of $f$ on $I$ if $f(c)$ is the least value for all $x$ in the interval (Absolute Minimum).

2. $f(c)$ is the maximum of $f$ on $I$ if $f(c)$ is the greatest value for all $x$ in the interval (Absolute Maximum).

Draw a graph with a minimum value. What is the value of $f'(c)$?

Draw a graph with a maximum value. What is the value of $f'(c)$?

**Extreme Value Theorem:** If $f$ is continuous on a **closed** interval $[a,b]$, then $f$ has both an **absolute** minimum and maximum value on the interval. These are called **extremum**.

**Critical Numbers:** Let $f$ be defined at $x = c$. If $f'(c) = 0$, or if $f'(c)$ does not exist, then $c$ is a critical number of $f$. 

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Relative Extrema: If $f$ has a relative minimum or maximum at $x = c$, then $c$ is a critical number of $f$. (**NOTE: Converse is not true.) Relative extrema do not occur at endpoints.

**NOTE:** Critical numbers **do not always** produce relative extrema.

Guidelines:
1. Differentiate the function and set $f'(x) = 0$. This will give you the critical values.
2. Evaluate $f(x)$ at the critical values.
3. Evaluate $f(x)$ at the endpoints.
4. The extreme values are the least and greatest at these points.

Guided Practice

Have students sketch the graph, and find the maximum and minimum values of:

1. $f(x) = 3x^4 - 4x^3$ \([-1, 2]\]

2. $f(x) = x^3 - 3x^2$ \([-1, 3]\]

3. $f(x) = 2x^2 - 3x^3$ \([-1, 3]\]

HW: P. 165 # 1 – 10, 11, 13, 14, 23, 26, 28, 42
Calculus I
Rolle’s Theorem, MVT

Objectives:
1. Students will graphically “discover” Rolle’s Theorem.
2. Students will use Rolle’s Theorem and Mean Value Theorem.

NCTM Principles and Standards
Geometry:
1. Analyze characteristics and properties of two-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
2. Specify locations and describe spatial relationships using coordinate geometry and other representational systems.
(Principles and Standards for School Mathematics Appendix 2000)

NYS Learning Standards for Mathematics
Geometry Strands:
1. Students will identify and justify geometric relationships formally and informally.
2. Use mathematics to show and understand physical phenomena.
(Mathematics Core Curriculum MST Standard 3 Revised March 2005)

Warm Up: True / False questions on overhead.

Have students graph \( f(x) = -x^2 + 2x + 3 \) on the interval \([-2,4]\) and answer the following questions.

1. What is the slope of the secant line between the endpoints?
2. What does a line with a slope equal to zero look like?
3. Is there any place within the interval where we could graph a horizontal tangent line?

Allow student discussion of \( f'(x) = 0 \), continuous functions, and importance of closed interval.
Rolles Theorem: Let $f$ be continuous on the closed interval $[a,b]$, and differentiable on the open interval $(a,b)$. If $f(a) = f(b)$, then there is at least one number $x = c$ in $(a,b)$ such that $f'(c) = 0$.

Have students graph $f(x) = x^4 - 2x^2$ on $[-2,2]$. Find all values of $c$ in the interval such that $f''(c) = 0$.

Have students graph $f(x) = 5 - \frac{4}{x}$ on $[1,4]$. Find the slope of the secant line between the endpoints. Ask students to determine if they could draw a tangent line parallel to the secant line within the interval. Allow discussion.

Mean Value Theorem: If $f$ is continuous on the closed interval $[a,b]$, and differentiable on the open interval $(a,b)$, then there is at least one number $x = c$ in $(a,b)$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

Guided Practice: P. 172 # 1 – 6, 10 – 12, 21 – 23, 26, 31 – 37 odd
Calculus I
Increasing, Decreasing intervals and the First Derivative Test

Objectives:
1. Students will determine intervals on which a function is increasing or decreasing.
2. Students will apply the first derivative test to find relative extrema.

NCTM Principles and Standards
Number and Operations: Understand numbers, ways of representing numbers, relationships among numbers, and number systems. (Principles and Standards for School Mathematics Appendix 2000)

NYS Learning Standards for Mathematics
Representation Strands: Students will create and use representations to organize, record, and communicate mathematical ideas. (Mathematics Core Curriculum MST Standard 3 Revised March 2005)

Warm Up:
1. Have students use algebra and calculus to find any relative extrema for \( f(x) = (x+5)^3 - 3 \) on the open interval \((-6, -4)\). Then have students graph \( f(x) = (x+5)^3 - 3 \) on the closed interval \([-6, -4]\).

2. Using the graph \( f(x) = -x^2 + 2x + 3 \) on the interval \([-2, 4]\), have students explain visual representation of increasing, decreasing, relative extrema, and what algebra or calculus support their conclusions. Where does the graph of \( f \) change from increasing to decreasing, or vice versa?

3. Have the same discussion for \( f(x) = x^4 - 2x^2 \) on \([-2, 2]\).

Ask: What is true about the slope of any increasing function?

Any decreasing function?

A function \( f \) is increasing on an interval if for any two numbers \( x_1 \) and \( x_2 \) in the given interval, \( x_1 < x_2 \) implies \( f(x_1) < f(x_2) \).
A function \( f \) is **decreasing** on an interval if for any two numbers \( x_1 \) and \( x_2 \) in the given interval, \( x_1 < x_2 \) implies \( f(x_1) > f(x_2) \).

**First derivative test for increasing and decreasing functions**

Let \( f \) be a function that is continuous on the closed interval \([a,b]\) and differentiable on the open interval \((a,b)\).

1. If \( f'(x) > 0 \) for all \( x \) in \((a,b)\), then \( f \) is increasing.
2. If \( f'(x) < 0 \) for all \( x \) in \((a,b)\), then \( f \) is decreasing.
3. If \( f'(x) = 0 \) for all \( x \) in \((a,b)\), then \( f \) is constant.

Lead students to an analytical understanding of the first derivative sign-change test to determine relative extrema using visual of graphs.

**Guided Practice**

1. Find the open intervals on which \( f(x) = x^3 - \frac{3}{2}x^2 \) is increasing or decreasing, and state any relative extrema.

2. Find the relative extrema of \( f(x) = \left(x^2 - 4\right)^{\frac{2}{3}} \)

3. If a projectile is propelled from ground level and air resistance is negligible, the object will travel farthest with an initial angle of \( 45^\circ \). If the projectile is propelled from a point above ground, the angle that yields maximum distance is not \( 45^\circ \).

The path of a projectile that is propelled at an angle \( \theta \) is

\[
y = \frac{g \sec^2 \theta}{2v_0^2} x^2 + (\tan \theta) x + h, \quad 0 \leq \theta \leq \frac{\pi}{2}
\]

\( y \) is the height, \( x \) is the horizontal distance, \( g \) is the acceleration due to gravity, \( v_0 \) is the initial velocity, and
$h$ is the initial height. Let $g = -32 \text{ ft/sec}$, $v_0 = 24 \text{ ft/sec}$, and $h = 9 \text{ ft}$. What value of $\theta$ will yield the maximum horizontal distance? (Hint: To find the distance the projectile travels, let $y = 0$ and use the quadratic formula to solve for $x$.)

P. 181 # 1 – 10, 13, 18, 19, 29, 33, 34, 36
Calculus I
Concavity and the Second Derivative Test

Objectives:
1. Students will determine intervals on which a function is concave up or concave down.
2. Students will apply the second derivative test to find points of inflection and relative extrema.

NCTM Principles and Standards
Number and Operations:
1. Create and use representations to organize, record, and communicate mathematical ideas.
2. Specify locations and describe spatial relationships using coordinate geometry and other representational systems.
(Principles and Standards for School Mathematics Appendix 2000)

NYS Learning Standards for Mathematics
Geometry Strands: Students will investigate and analyze characteristics and properties of geometric shapes.
(Mathematics Core Curriculum MST Standard 3 Revised March 2005)

Warm Up: True / False questions on overhead.

Let $f$ be differentiable on an open interval $I$. The graph of $f$ is **concave upward** on $I$ if $f'$ is increasing on the interval and **concave downward** on $I$ if $f'$ is decreasing on the interval.
On the given graph, draw several tangent lines. What do you notice about the placement of the tangent line where the graph is concave up? Concave down? What do you notice about the slope of the tangent line where the graph is concave up? Concave down? (Allow for discussion that covers the topic of the meaning of $f'$ increasing or decreasing.)

We have previously used the sign of the derivative $f'$ to determine where a function $f$ was increasing and where it was decreasing. Similarly, we can use the sign of the second derivate $f''$ to determine where the first derivative $f'$ is increasing and where it is decreasing. Specifically, we are interested when $f'' > 0$ on an open interval $I$, then $f'$ (x) is increasing on $I$; the slope of the tangent line to the graph of $f$ increases as $x$ increases.

$$f''(x) > 0$$
$$f'$$ increasing $\rightarrow$ concave up

$$f''(x) < 0$$
$$f'$$ decreasing $\rightarrow$ concave down

Example 1: $f(x) = x^3 + x^2 - 5x - 5$

Determine intervals on which the graph of $f$ is concave up or is concave down, and illustrate the results graphically.

Is $f$ continuous? (yes) Is $f$ differentiable? (yes)

Find the second derivative: $f''(x) = 6x + 2$

Set the second derivative $= 0$: $6x + 2 = 0$  $2(3x+1) = 0$  $x = \frac{-1}{3}$

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<td>upward</td>
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**Points of Inflection:**
The point on the graph where the concavity changes is called the point of inflection.

Each point on the graph of \( f \) at which the concavity changes from upward to downward, or vice versa, is called a point of inflection.

Definition: A point \( (c, f(c)) \) on the graph of \( f \) is a point of inflection if the following two conditions are satisfied:
1. \( f \) is continuous at \( c \).
2. There is an open interval \((a, b)\) containing \( c \) such that the graph is concave up on \((a, c)\), and concave down on \((c, b)\), or vice versa.

**The second derivative Test:**
We can use the second derivative to test for concavity, and we can also use it to find relative minima and maxima. (If the graph of \( f \) is concave up on an open interval containing \( c \), and \( f''(c) = 0 \), \( f(c) \) must be a relative minimum of \( f \).)

Similarly, if the graph of a function \( f \) is concave downward on an open interval containing \( c \), and \( f''(c) = 0 \), \( f(c) \) must be a relative maximum of \( f \).

Let \( f \) be continuous, \( f'(c) = 0 \), and the second derivative exists on open interval containing \( c \):
1. If \( f''(c) > 0 \), then \( f(c) \) is a relative minimum.
2. If \( f''(c) < 0 \), then \( f(c) \) is a relative maximum.
3. If \( f''(c) = 0 \), then the test fails. Use the first derivative test.

Example: Find the relative extrema for \( f(x) = -3x^5 + 5x^3 \)

First, find the critical numbers by setting the first derivative = 0:

\[
f'(x) = -15x^4 + 15x^2
\]

\[-15x^4 + 15x^2 = 0 \quad \Rightarrow \quad 15x^2(1-x^2) = 0 \quad \Rightarrow \quad \text{Critical numbers: } x = 0, \ x = \pm 1
\]

\[
f''(x) = -60x^3 + 30x \quad \Rightarrow \quad f''(x) = 30(-2x^3 + x)
\]

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<td>Relative maximum</td>
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HW: P. 189 # 1 – 10, 11, 14, 16
Calculus I
Creating Graphs

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Objectives:
1. Students will graph a given set of equations.
2. Students will perform transformations to the given graphs to create a design.

NCTM Principles and Standards
Representation Standard: Use representations to model and interpret physical, social, and mathematical phenomena.
Connections Standard: Students will increase their abilities to visualize, describe, and analyze in mathematical terms.
Geometry Standards:
1. Analyze properties and determine attributes of two- and three-dimensional objects.
2. Understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, and function notation.
(Principles and Standards for School Mathematics Appendix 2000)

NYS Learning Standards for Mathematics
Representation Strand: Investigate relationships between different representations and their impact on a given problem.
Geometry Strand: Students will use visualization and spatial reasoning to analyze characteristics and properties of geometric shapes.

Problem-Solving Strands:
1. Observe and explain patterns to formulate generalizations and conjectures.
2. Use a variety of problem-solving strategies to understand new mathematical content.
3. Students will apply transformations and symmetry to analyze problem-solving situations.
(Mathematics Core Curriculum MST Standard 3 Revised March 2005)

Overview:
Say to Students: “Remember in 8th grade? Remember the cool cartoon pictures we made on the coordinate plane by connecting points of ordered pairs with line segments? Well, we are going to introduce a more sophisticated method of creating art. Using curve-sketching techniques, we will “adjust” a given set of functions to create a new picture.”

Directions: Graph the following functions and relations on the given domain. Then modify each equation to create a new image. Explain each modification
and purpose for the modification. Include in the discussion any use of first or second derivative, points of inflection, or increasing or decreasing intervals.

1. \( y = \cos(3x - 1.2) + 1 \), \([-5, 3.2]\)
2. \( y = \sin(3x - 1.2) + 3 \), \([-5, 3.2]\)
3. \( 2(x - 1.2)^2 + (y - 2.3)^2 = 1 \)
4. \( (x - 1.2)^2 + (y - 2.3)^2 = .07 \)

Student work will vary. This assignment gives students the opportunity to investigate transformations of a given set of graphs. Students also have the opportunity to explore unique transformations and to create an individual design.
Calculus I
Creating Graphs – The Butterfly

Objectives:
3. Students will graph a given set of equations to make a picture.
4. Students will analyze the graphs and equations.

NCTM Principles and Standards
The Learning Principle: Students will learn mathematics with understanding, actively building new knowledge based on previous experience and on prior knowledge. (Principles and Standards for School Mathematics Appendix 2000)

NYS Learning Standards for Mathematics
Geometry Strands: Students will identify and justify geometric relationships formally and informally. Students will apply transformations and symmetry to analyze problem-solving situations.

Directions: Using sophisticated graphs, students will graph a given set of functions to create a butterfly (by Natalie Brubaker, Calculus I). Students will then answer a series of questions based on the graph.

The following graphs are given with a specific interval. Students will use a graphing program to create the following image.

1. \( y = -.3 \log(-x - 1.3) \)  
2. \( y = -2(x + 2)^2 + 5.55 \)  
3. \( y = 2.5 \sin(0.7x - 15) \)  
4. \( y = 0.4x - 1 \)  
5. \( y = \sin(x - 4) - .55 \)  
6. \( y = \arctan(2x) \)  
7. \( y = \sin(x + 2) - 1.33 \)  
8. \( y = -\sin(x + 1) -.5 \)  
9. \( y = \sin(x + 1) - 2 \)  
10. \( y = x - 1 \)  
11. \( y = -x - 3.5 \)  
12. \( y = \arccos(x) - 1 \)  
13. \( y = \arctan(3x - 1.25) + .5 \)  
14. \( y = \arctan(3x - 1.5) + 1 \)  
15. \( .1 = (x - .35)^2 + (y - .35)^2 \)  
16. \( .03 = (x - .6)^2 + (y - .35)^2 \)  
17. \( (x - .84)^2 + (y + 1)^2 = .02 \)  
18. \( (-1.32, .5), (-1.33, 4.55) \)
1. Discuss the connection between \( y = \sin(x - 4) - 0.55 \) on \([-3.33, 0.087]\) and \( y = \arctan(2x) \) on \([-3.2, 0.087]\). Why do these two functions have different ranges?

2. Can you think of another function that could replace \( y = 2.5 \sin(0.7x - 15) \) on \([-5.63, -1.32]\)?

3. Describe the transformations of the graph \( y = \cos(x) \) to create the left antenna.
Creating Graphs Project

Students will create a design(s) made of a series of functions and relations. After sketching a design on the coordinate plane, students will develop functions and set the intervals to depict their sketch.

NCTM: Technology supporting learning. Student’s learning is enhanced by the use of technology. Standards: Geometry, analysis, and representation. (Principles and Standards for School Mathematics Appendix 2000)


This lesson focuses on meeting the challenge of “disengagement” of student interest by extending student learning experience and combining it with a creative opportunity. The goal is to engage otherwise uninterested students with a nontraditional assignment that requires demonstration of conceptual understanding and problem solving.

Evaluative Process:

1. Students will submit sketched design on the coordinate plane.
2. Design will be broken into segments. Each segment will have a function or relation and a given interval associated with it.
3. Each submitted sketch will have a minimum of eight curves.
4. Each submission will contain at least 4 different “types” of curves; trig, logarithmic, quadratic, elliptical, linear, exponential, ...
5. Students will submit a final design created using a graphing program (downloads from calculators, WinPlot, Graphmatica, Sketchpad,... Each design will be accompanied by the list of curves and the intervals they are graphed on.
6. Students may submit multiple designs.

Designs will be graded on accuracy of graphs to portray the original sketch. Sketches may be modified slightly.

Happy Graphing!!

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