Faculty and Staff Article:

Yong Q. An and Alain C. Diebold

Colleges of Nanoscale Science and Engineering, SUNY Polytechnic Institute

Citation for Article:

An, Y. Q., & Diebold, A. C. (2017). Transiently stimulated second-harmonic generation from silicon nanogratings. *Physical Review B*, 96(20), 201306. doi: 10.1103/PhysRevB.96.201306

<https://journals.aps.org/prb/abstract/10.1103/PhysRevB.96.201306>

Transiently stimulated second-harmonic generation from silicon nanogratings

Yong Q. An* and Alain C. Diebold

Colleges of Nanoscale Science and Engineering, SUNY Polytechnic Institute, Albany, New York 12203, USA

(Received 7 March 2017; revised manuscript received 8 November 2017; published 29 November 2017)

We observe that second-harmonic (SH) light generated by a femtosecond probe pulse from Si nanograting surfaces can be enhanced by a correlating pump pulse. Time-resolved anisotropy analysis shows that the enhancement arises from transient stimulation by the pump because it polarizes the periodically distributed charge from the probe photoinjection. In addition, dipole and quadrupole SH components are stimulated differently depending on the pump polarization. This effect reveals ultrafast SH dynamics in centrosymmetric materials and provides guidance for measuring nanoscale features.

DOI: [10.1103/PhysRevB.96.201306](https://doi.org/10.1103/PhysRevB.96.201306)

Recently, it has been demonstrated that silicon can be used as an unconventional medium for high-harmonic generation [1], suggesting its broader applications in the fields of nonlinear optics and photonics. This was achieved by using the plasmonic effects of periodic metallic nanostructures [2] to localize the fundamental light into subwavelength regions on a planar Si substrate. Si nanogratings should be a more efficient medium for such applications because they have a periodicity (pitch) small enough to prevent diffraction at extreme ultraviolet wavelengths and thus may more tightly localize both light waves and static fields into nanoscale regions. However, there have been very few experiments conducted to test this concept. Femtosecond time-resolved second-harmonic generation (SHG) is a powerful probe for ultrafast dynamics in centrosymmetric materials [3,4], and here we perform such an experiment on Si nanogratings to study the physics and dynamics of harmonic emission. We find that SHG from nanoscale Si can be coherently enhanced by optical stimulation in a noncollinear pump-probe configuration [5], similar to the effect of stimulated Raman scattering [6,7].

In a pump-probe experiment, a pump pulse excites the sample into a nonequilibrium state, and a probe pulse tracks the rise and decay of the excitation by detecting the signal change as a function of pump-probe delay time. This technique has been used to study ultrafast phenomena in semiconductors, such as carrier dynamics [8,9], charge transfer and trapping [10], and lattice dynamics [11,12]. Although the pump is responsible for carrier excitation, some excitation from the probe is hard to avoid, especially in pump-probe SHG experiments, where the probe SHG requires intense pulses for sufficient signals. Successive excitations by multiple probe pulses may cause accumulation and trapping of photoinjected charge, resulting in a slow variation of SHG over the measurement duration because a time-dependent dc electric field is introduced at the surface to produce electric-field-induced SHG (FI-SH) [13–15]. Here we observe that charge photoinjection in Si nanogratings can give rise to a new femtosecond pump-probe effect, namely, transiently stimulated SHG, in which a weak pump enhances the SHG by a strong probe.

Previous pump-probe SHG experiments have been performed mainly on noncentrosymmetric materials that allow dipole SHG in the bulk, such as GaAs or SrTiO₃ bulk crystals

[12,16,17] and PbSe nanocrystals that were randomly oriented in a thin film [10]. Nanoscale centrosymmetric crystals with well-defined crystalline orientations, such as Si nanogratings, have not been studied mostly because pump-probe effects were weak or samples were not available. However, it is desirable to perform such a study to understand the physics and dynamics of electric quadrupole SHG [18–21], which is an important higher order component that is often buried in stronger dipole SHG signals. Nanograting structures with an array of fins, as sketched and imaged in Fig. 1(a), have been patterned on a Si wafer by advanced lithographic techniques. They are used as the body of fin field-effect transistors, which are the building blocks of nanoelectronic devices. They may be integrated into photonic systems to realize desired optical functionalities, such as electrically or optically controlled frequency conversion. Here we perform pump-probe SHG and anisotropy experiments on Si nanogratings to separately determine the dynamics of surface dipole SHG and bulk quadrupole SHG. We observe that transiently stimulated SHG strongly depends on the pump polarization and sample azimuth.

In this experiment, we used pulses from a Ti:S laser with the pulse duration of 120 fs, repetition rate of 82 MHz, and wavelength of 740 nm. The laser beam was split into a pump and a probe, and then they overlapped on the sample. The pump beam, at a 30° incident angle, was focused to a 75-μm spot. The probe beam, at a 45° incident angle, was focused to a 30-μm spot. The typical fluence of the probe was 6.25 times stronger than that of the pump. The experimental configurations, including the pump, probe, and SHG beams with polarization, and the Si nanograting sample with crystal orientations, are shown in Fig. 1(a). The pump-probe delay time was controlled by a translation stage. SH photons were collected by a photomultiplier. Here we mainly show the results from a nanograting with a pitch of 42 nm and a height of 58 nm. These features are exhibited by the scanning electron microscopy (SEM) image of the nanograting cross section in Fig. 1(a). The nanogratings were fabricated on undoped Si(001) wafers by high-resolution lithography.

Figure 1(b) shows the measured SH transients from a 42-nm-pitch nanograting sample under either a *p*- or *s*-polarized pump, showing the SH signal as a function of the pump-probe delay time *t*. The grating lines are aligned parallel to the incident plane, and both the pump and probe powers are fixed at 120 mW. Under the *p*-polarized pump, the SH transient

*yan2@sunypoly.edu

FIG. 1. (a) Pump-probe SHG geometry showing the laser beams with polarizations, the Si nanograting, and the Si(001) substrate with crystalline orientations. The inset shows a cross-section SEM image of a 42-nm-pitch silicon nanograting. (b) Measured pp -polarized SH transients with a p - or s -polarized pump when the grating line is in the incident plane. Smooth lines are Eqs. (1) and (2) fits at two different stages, respectively. (c) Measured pp -polarized SH transients with a p - or s -polarized pump from a planar Si(001) surface. Smooth lines are Eq. (3) fits.

shows a large peak, containing an initial rise within 0.2 picosecond (ps), a subsequent relaxation within 0.2 ps, and a continuing relaxation containing damped oscillations within 0.4 ps. During the entire time range, the pump-induced change of SHG as a function of the pump-probe delay time can be fit by the function

$$\Delta I_1^{(2\omega)} = \Delta I_0^{(2\omega)} e^{-2(\frac{t}{T})^2}. \quad (1)$$

This derives from the intensity autocorrelation of two pulses, each with a Gaussian temporal profile. Here, $\Delta I_0^{(2\omega)}$ is the amplitude, and T the correlation time constant. The Eq. (1) fit in Fig. 1(b) shows $T = 237$ fs, corresponding to a pulse duration of 160 fs. When $t \geq 0.15$ ps, a correction term can be added to Eq. (1) to represent the oscillating tail of relaxation, so the pump-induced change of SHG becomes

$$\Delta I_2^{(2\omega)} = \Delta I_1^{(2\omega)} + I_{\text{osc}}^{(2\omega)} e^{-t/\tau_{\text{osc}}} \cos(2\pi f t). \quad (2)$$

Here, $I_{\text{osc}}^{(2\omega)}$ is the initial amplitude of oscillation, f is the oscillation frequency, and τ_{osc} is the oscillation decay time constant. Figure 1(b) shows that the p -polarized pump enhances SHG by 25%, but the s -polarized pump barely has any effect, both on the signal level and the transient change. The observed short correlation time, as corroborated by its polarization dependence, indicates that the pump-probe interaction is a coherent nonlinear process. The oscillations in the SH transient have a frequency of $f = 4.5$ THz, corresponding to a transverse acoustic mode of photoexcited phonons in Si [22].

Figure 1(c) shows the measured SH transients from a planar Si(001) surface with native oxide termination under the same experimental conditions as above. Both the p - and s -polarized pumps produce almost the same SH transient, containing a fast drop of the SH signal within 0.1 ps and a continuing slow drop within 2.5 ps. The time for the relaxation has not been determined in this experiment, but is estimated to be hundreds of ps based on previous pump-probe reflectivity results [8]. The slow drop of the SH signal can be fit by a decaying exponential

$$\Delta I^{(2\omega)} = \Delta I_0^{(2\omega)} (1 - e^{-\frac{t}{\tau}}). \quad (3)$$

Here, $\Delta I_0^{(2\omega)}$ is the drop amplitude and τ is the time constant for the excitation. Equation (3) fits in Fig. 1(c)

indicate $\tau \approx 0.8$ ps, which is four times longer than the number obtained from linear optics experiments [8]. This is reasonable considering that SHG tends to access slower dynamics involving trapped charge at the surface. The slow excitation together with its pump polarization independence indicates that the pump-probe interaction is an incoherent process [23,24], involving excitation of electron-hole pairs and subsequent recombination with trapped charge at the surface.

To identify the observed SH dynamics, we performed rotational-anisotropy SHG (RA-SH) measurements by recording the SH signal as a function of sample azimuthal angle Φ , which is between the incident plane and the grating lines. Different polarization configurations are used to emphasize on different SH components. The pump, probe beam, and the SH analyzer can be either p - or s -polarized. Here ps denotes a configuration with p -polarized fundamental light and s -polarized SH analyzer, etc. Figure 2(a) shows the pp -polarized RA-SH patterns measured consecutively for three scans on a fresh sample spot that has not been exposed to the laser. The laser power was 120 mW. Over the first three scans, the SH signal increases about 100%. Such strong time dependence arises from laser-induced charge photoinjection and trapping at the surface, which introduces a FI-SH effect. Figure 2(b) shows the pp -polarized RA-SH scans measured with or without a pump. The pump is either p - or s -polarized, and the delay is fixed at $t = 0$ ps. These scans were taken when the SH signals had reached saturation. Figures 2(c) and 2(d) show the ps -polarized RA-SH scans measured under otherwise the same conditions as in Figs. 2(a) and 2(b), respectively.

We can understand the observed SH anisotropy based on symmetry considerations for the two main SH contributions: dipole SHG at the nanograting surface and quadrupole SHG in the Si bulk. The surface is macroscopically flat, so dipole SHG can be described by a third rank susceptibility tensor $\chi_{ijk}^{(2)}$. Because of twofold symmetry, this surface tensor has only five independent tensor elements χ_{zxx} , χ_{zyy} , χ_{xxz} , χ_{yyz} , and χ_{zzz} . Here, x is along the grating line and z is in the surface normal. In the bulk of Si, dipole SHG is forbidden, but quadrupole SHG is allowed and can be described by a fourth rank tensor. Because of cubic symmetry, the bulk tensor has only two independent elements, ζ and γ [25,26].

FIG. 2. Measured *pp*-polarized SH anisotropy scans from a Si nanograting: (a) on a fresh spot continuously for three scans; (b) on an irradiated spot without a pump and with a pump of *p*- or *s* polarization. Measured *ps*-polarized SH anisotropy scans under the above conditions are shown in (c) and (d), respectively. Smooth lines are Eqs. (4) or (6) fits.

The *pp*-polarized RA-SH patterns from the nanograting can be written as

$$I_{pp}^{(2\omega)} = [a_{0,pp} + a_{2,pp} \cos(2\phi) + a_{4,pp} \cos(4\phi)]^2, \quad (4)$$

where the coefficients are linear functions of related tensor elements. Specifically,

$$\begin{aligned} a_{2,pp} &= c_{21,pp}(\chi_{zxx} - \chi_{zyy}) + c_{22,pp}(\chi_{xxz} - \chi_{yyz}); \\ a_{4,pp} &= c_{4,pp}\zeta, \end{aligned} \quad (5)$$

where c_{21} , c_{22} , and c_4 are proportional to the probe power and a constant determined by the dielectric function of Si. Therefore $a_{2,pp}$ is determined only by dipole SHG, and $a_{4,pp}$ only by quadrupole SHG. The coefficient $a_{0,pp}$ is a linear function of all seven tensor elements, but ζ and γ should contribute negligibly [26], and therefore, $a_{0,pp}$ is mainly determined by dipole SHG. In Fig. 2(a), the Eq. (4) fit values of both $a_{0,pp}$ and $a_{2,pp}$ dominant over $a_{4,pp}$, meaning that the $a_{4,pp}$ value is trivial and thus hard to extract. All three coefficients $a_{0,pp}$, $a_{2,pp}$, and $a_{4,pp}$ consistently grow over the initial three scans, suggesting that the FI-SH can be described by a susceptibility tensor $E^{dc} \chi_{ijk}^{(2)}$, where E^{dc} is the dc electric field built by photoinjection at the surface. In Fig. 2(b), the *s*-polarized pump raises both $a_{0,pp}$ and $a_{2,pp}$ but reduces $a_{4,pp}$, while the *p*-polarized pump raises all three coefficients. This explains the observation in Fig. 1(b) that the SH transient under the *s*-polarized pump is flat, because the changes on different SH components cancel each other at $\phi = 0^\circ$. The *s*-polarized pump indeed changes the RA-SH shape, as evidenced in Fig. 2(b).

The *ps*-polarized RA-SH patterns from the nanograting can be written as

$$I_{ps}^{(2\omega)} = [a_{0,ps} + a_{2,ps} \sin(2\phi) + a_{4,ps} \sin(4\phi)]^2, \quad (6)$$

where the coefficients are linear functions of related tensor elements,

$$a_{0,ps} = 0; \quad a_{2,ps} = c_{22,ps}(\chi_{xxz} - \chi_{yyz}); \quad a_{4,ps} = c_{4,ps}\zeta. \quad (7)$$

Now dipole SHG is suppressed because $a_{0,ps} = 0$, and therefore, quadrupole SHG or $a_{4,ps}$ is relatively magnified. This situation benefits the evaluation of $a_{4,ps}$. The strength of $a_{4,ps}$ can now be visualized, simply by inspecting the RA-SH shape. More deviation off 90° for the gap between two adjacent RA-SH peaks means more strength of $a_{4,ps}$. Figure 2(c) shows $a_{0,ps} \neq 0$ because the RA-SH amplitudes modulate, but this observation contradicts the predicted $a_{0,ps} = 0$ in Eq. (7). Figure 2(d) shows that pumps of both polarizations change the RA-SH pattern, but the *p*-polarized pump is more effective. In addition, the RA-SH peaks are shifted differently for *p*- and *s*-polarized pumps, indicating that the three fit coefficients are not changed proportionally.

The inconsistency between the observed $a_{0,ps} \neq 0$ in Fig. 2 and the predicted $a_{0,ps} = 0$ in Eq. (7) is caused by uneven history of photoinjection on the RA-SH scanning trace, which is almost inevitable if the scanning diameter of the beam trace ring is tiny. In the case of Fig. 2, the scanning diameter

FIG. 3. (a) Measured ps -polarized SH anisotropy at different probe powers of 288, 144, 72, and 36 mW. (b) Measured ps -polarized SH anisotropy at a fixed probe power of 122 mW without a pump and with a p - or s -polarized pump. Smooth lines are Eq. (6) fits. (c) Normalized fit coefficients, a_2 and a_4 , as a function of the probe power. These coefficients with a p - or s -polarized pump are also plotted.

was $60 \mu\text{m}$. Before each RA-SH measurement, the laser had irradiated one side of the scanning region longer than the other side, causing uneven FI-SH that appears as a onefold symmetry resulting in $a_{0,ps} \neq 0$. To verify this explanation, we enlarged the RA-SH scanning area so that the unevenness of FI-SH can be removed. Figure 3(a) shows the measured ps -polarized RA-SH scans when the scanning diameter was $600 \mu\text{m}$. The observed $a_{0,ps} = 0$ is evidenced by almost the same peak amplitudes, and this consistency is held for laser powers of 288, 144, 72, and 36 mW. However, the RA-SH peak shift becomes larger as laser power increases, indicating that quadrupole SHG gains more strength at higher laser powers, relative to dipole SHG.

Figure 3(b) shows the ps -polarized RA-SH scans measured using a $600\text{-}\mu\text{m}$ scanning diameter with and without a pump. The pump, either p - or s -polarized, is at a fixed delay $t = 0$ ps. With a pump, the observed $a_{0,ps} \neq 0$ can be explained by anisotropic absorption of the pump. Figure 3(c) summarizes the Eq. (6) fit results of the coefficients $a_{2,ps}$ and $a_{4,ps}$ both in Figs. 3(a) and 3(b). At high laser power, both $a_{2,ps}/I^{(\omega)}$ and $a_{4,ps}/I^{(\omega)}$ increase as the power increases, while in the case of intrinsic susceptibilities they would be constant. At low laser power, $a_{2,ps}/I^{(\omega)}$ approaches a constant as the power decreases, while $a_{4,ps}/I^{(\omega)}$ can be extrapolated to near zero. These results show that photoinjection affects dipole SHG only at high powers, but affects quadrupole SHG at all powers. The power dependence indicates that quadrupole SHG originates from the FI-SH of photoinjection. At a fixed power of 120 mW, the p -polarized pump raises both $a_{2,ps}$ and $a_{4,ps}$, while the s -polarized pump raises $a_{2,ps}$ but lowers $a_{4,ps}$.

This pump polarization dependence agrees on the results both in Figs. 2(b) and 2(d), confirming that an s -polarized pump enhances dipole SHG but diminishes quadrupole SHG.

We can understand the SH transients in Fig. 1 by drawing three different charge polarizations. In a nanograting, a p -polarized pump excites vertical dipoles of Fig. 4(a), which enhances both dipole and quadrupole SHG and thus results in a transient rise within 0.2 ps. An s -polarized pump excites horizontal dipoles of Fig. 4(b), which enhances dipole SHG but diminishes quadrupole SHG and thus results in a flat SH transient. On a planar Si surface, a pump excites electron-hole pairs, as shown in Fig 4(c), instead of dipoles because of the uniform charge distribution. These pairs recombine with negative charge at the surface, causing a SH drop within 2.5 ps

FIG. 4. Schematic dynamics of the pump-probe SHG from the Si nanograting for a p -polarized pump (a) and an s -polarized pump (b), and from the planar Si substrate (c).

as FI-SH lowers. We regard the observed pump-probe effect as transiently stimulated SHG based on the characteristics, as demonstrated in the coherent nonlinear process. Further evidence in support of the dynamics in Fig. 4 is that both the time-dependence and pump-probe SHG effects would virtually disappear if we use Si nanogratings with a 19-nm-thick Si_3N_4 tip (without intermediate SiO_2) for the experiments. This happens because the Si_3N_4 tip suppresses charge photo-injection and trapping and means surface charges at the tips are responsible for the SH dynamics.

To correlate SHG to nanoscale features, we performed the same type of measurements on a set of nanogratings with different pitches. The SH signals and RA-SH shapes can vary considerably with both measurement history and laser intensity, and this makes it difficult to find a consistent trend. To remove these factors, we consider the ratios among different fit coefficients. The sp -polarized RA-SH scans can be fit to Eq. (4) using the coefficients $a_{0,sp}$, $a_{2,sp}$, and $a_{4,sp}$. As $a_{4,sp}$ is negligible, the ratio of $a_{2,sp}$ and $a_{0,sp}$ takes the simple form

$$\frac{a_{2,sp}}{a_{0,sp}} = \frac{\chi_{zyy} - \chi_{zxx}}{\chi_{zyy} + \chi_{zxx}}, \quad (8)$$

but is most efficient for measuring the pitch size. For nanogratings with pitches of 63, 42, and 28 nm, we found $|a_{2,sp}/a_{0,sp}| = 0.46, 0.49, \text{ and } 0.76$, respectively. These ratios, for their respective pitches, have been reasonably consistent among different samples of similar grating heights, and this trend follows the Eq. (8) prediction that smaller pitches

correspond to more SH anisotropy. Transiently stimulated SHG also appears stronger in nanogratings with smaller pitches, and this trend can be understood by the fact that transient polarization is proportional to the number of vertical dipoles. We hope to obtain nanogratings with smaller pitches (< 28 nm) to confirm these trends of variation and find their limits.

In summary, we have discovered a new femtosecond pump-probe SHG effect on technologically important Si nanogratings: a correctly polarized weak pump pulse can coherently enhance the SH signal generated by a strong probe pulse. We identified this effect as transiently stimulated SHG, unique to nanograting surfaces, in which the periodically distributed charge from probe-injection is polarized by the pump. We found that quadrupole SHG in Si arises from charge photo-injection and can be enhanced or diminished depending on the pump polarization. These results suggest new SHG techniques for measuring nanoscale features. The transient SHG effect can be exploited to modulate SHG at ultrafast optical speed on a Si platform, to manipulate light polarization at extreme ultraviolet wavelengths, and to generate high harmonic light by using Si nanogratings when capped with plasmonic metals or interfaced with noble gases.

We thank Gangadhara Muthinti and Dhairya Dixit for sample preparation, SEM and XRD characterization, and helpful discussions. We thank Avery Green and Luke Blanco for critical reading and making comments on the paper. We acknowledge funding from SRC GRC under task 2337.002.

-
- [1] G. Vampa, B. G. Ghamsari, S. Siadat Mousavi, T. J. Hammond, A. Olivieri, E. Lisicka-Skreke, A. Y. Naumov, D. M. Villeneuve, A. Staudte, P. Berini, and P. B. Corkum, *Nat. Phys.* **13**, 659 (2017).
 - [2] A. Husakou, S. J. Im, and J. Herrmann, *Phys. Rev. A* **83**, 043839 (2011).
 - [3] Y. R. Shen, *Nature* **337**, 519 (1989).
 - [4] P. J. Nowakowski, D. A. Woods, C. D. Bain, and J. R. R. Verlet, *J. Chem. Phys.* **142**, 084201 (2015).
 - [5] A. J. Goodman and W. A. Tisdale, *Phys. Rev. Lett.* **114**, 183902 (2015).
 - [6] F. Benabid, J. C. Knight, G. Antonopoulos, and P. S. J. Russell, *Science* **298**, 399 (2002).
 - [7] C. W. Freudiger, W. Min, B. G. Saar, S. Lu, G. R. Holtom, C. He, J. C. Tsai, J. X. Kang, and X. S. Xie, *Science* **322**, 1857 (2008).
 - [8] A. J. Sabbah and D. M. Riffe, *Phys. Rev. B* **66**, 165217 (2002).
 - [9] M. Maurer, I. L. Shumay, W. Berthold, and U. Höfer, *Phys. Rev. B* **73**, 245305 (2006).
 - [10] W. A. Tisdale, K. J. Williams, B. A. Timp, D. J. Norris, E. S. Aydil, and X.-Y. Zhu, *Science* **328**, 1543 (2010).
 - [11] C. V. Shank, R. Yen, and C. Hirlimann, *Phys. Rev. Lett.* **51**, 900 (1983).
 - [12] Y. M. Chang, L. Xu, and H. W. K. Tom, *Phys. Rev. Lett.* **78**, 4649 (1997).
 - [13] J. I. Dadap, P. T. Wilson, M. H. Anderson, M. C. Downer, and M. terBeek, *Opt. Lett.* **22**, 901 (1997).
 - [14] Y. Q. An, J. Price, M. Lei, and M. C. Downer, *Appl. Phys. Lett.* **102**, 051602 (2013).
 - [15] A. L. Chekhov, I. Razdolski, A. Kirilyuk, T. Rasing, A. I. Stognij, and T. V. Murzina, *Phys. Rev. B* **93**, 161405 (2016).
 - [16] Y. D. Glinka, T. V. Shahbazyan, I. E. Perakis, N. H. Tolk, X. Liu, Y. Sasaki, and J. K. Furdyna, *Appl. Phys. Lett.* **81**, 220 (2002).
 - [17] E. D. Mishina, T. V. Misurayev, N. E. Sherstyuk, V. V. Lemanov, A. I. Morozov, A. S. Sigov, and T. Rasing, *Phys. Rev. Lett.* **85**, 3664 (2000).
 - [18] P. Guyot-Sionnest and Y. R. Shen, *Phys. Rev. B* **38**, 7985 (1988).
 - [19] N. Bloembergen, R. K. Chang, S. S. Jha, and C. H. Lee, *Phys. Rev.* **174**, 813 (1968).
 - [20] H. W. K. Tom, T. F. Heinz, and Y. R. Shen, *Phys. Rev. Lett.* **51**, 1983 (1983).
 - [21] J. I. Dadap, J. Shan, K. B. Eisenthal, and T. F. Heinz, *Phys. Rev. Lett.* **83**, 4045 (1999).
 - [22] M. Methfessel, *Phys. Rev. B* **38**, 1537 (1988).
 - [23] P. Borri, F. Romstad, W. Langbein, A. E. Kelly, J. Mørk, and J. M. Hvam, *Opt. Express* **7**, 107 (2000).
 - [24] S. L. Palfrey and T. F. Heinz, *J. Opt. Soc. Am. B* **2**, 674 (1985).
 - [25] J. E. Sipe, D. J. Moss, and H. M. van Driel, *Phys. Rev. B* **35**, 1129 (1987).
 - [26] Y. Q. An, R. Carrioles, and M. C. Downer, *Phys. Rev. B* **75**, 241307(R) (2007).